Nonlinear Analysis: Real World Applications 25 (2015) 96–111

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Well-posedness for dislocation based gradient visco-plasticity with isotropic hardening

N. Kraynyukova[a,1], S. Nesenenko[b,*], P. Neff[c], K. Chełmiński[d,2]

[a] *Fachbereich Mathematik, Technische Universität Darmstadt, Schlossgartenstrasse 7, 64289 Darmstadt, Germany*

[b] *Fakultät für Mathematik, Universität Duisburg–Essen, Campus Essen, Thea-Leymann-Straße 9, 45127 Essen, Germany*

[c] *Lehrstuhl für Nichtlineare Analysis und Modellierung, Fakultät für Mathematik, Universität Duisburg–Essen, Campus Essen, Thea-Leymann-Straße 9, 45127 Essen, Germany*

[d] *Faculty of Mathematics and Information Science, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland*

ARTICLE INFO

Article history:
Received 5 September 2014
Received in revised form 5 March 2015
Accepted 11 March 2015
Available online 2 April 2015

Keywords:
Gradient viscoplasticity
Rate dependent response
Subdifferential flow rule
Rothe's time-discretization method
Maximal monotone method
Geometrically necessary dislocations
Plastic spin
Defect energy

ABSTRACT

In this work we establish the well-posedness for infinitesimal dislocation based gradient viscoplasticity with isotropic hardening for general subdifferential plastic flows. We assume an additive split of the displacement gradient into non-symmetric elastic distortion and non-symmetric plastic distortion. The thermodynamic potential is augmented with a term taking the dislocation density tensor $\operatorname{Curl} p$ into account. The constitutive equations in the models we study are assumed to be of self-controlling type. Based on the self-controlling property the existence of solutions of quasi-static initial–boundary value problems under consideration is shown using a time-discretization technique and a monotone operator method.

1. Introduction

Within the framework of the strain gradient plasticity theory we study the existence and uniqueness of solutions of quasistatic initial–boundary value problems arising in gradient viscoplasticity with isotropic hardening. The models we study are introduced in [1] and use rate-dependent constitutive equations with internal variables to describe the deformation behaviour of metals at infinitesimally small strain.

* Corresponding author.
E-mail addresses: kraynyukova@mathematik.tu-darmstadt.de (N. Kraynyukova), sergiy.nesenenko@uni-due.de (S. Nesenenko), patrizio.neff@uni-due.de (P. Neff).

1 Tel.: +49 6151 16 3287.

2 Tel.: +49 201 183 4243.

http://dx.doi.org/10.1016/j.nonrwa.2015.03.004
1468-1218/

Setting of the problem. Let $\Omega \subset \mathbb{R}^3$ be an open bounded set, the set of material points of the solid body, with a C^1-boundary $\partial\Omega$. By T_e we denote a positive number (time of existence), which can be chosen arbitrarily large, and for $0 < t \le T_e$

$$\Omega_t = \Omega \times (0, t).$$

The sets, $\mathcal{M}^3$ and $\mathcal{S}^3$ denote the sets of all 3×3-matrices and of all symmetric 3×3-matrices, respectively. We recall that the space of all 3×3-matrices $\mathcal{M}^3$ can be isomorphically identified with the space $\mathbb{R}^9$. Therefore we can define a linear mapping $B : \mathbb{R}^N \to \mathcal{M}^3$ as a composition of a projector from $\mathbb{R}^N$ onto $\mathbb{R}^9$ and the isomorphism between $\mathbb{R}^9$ and $\mathcal{M}^3$. The transpose $B^T : \mathcal{M}^3 \to \mathbb{R}^N$ is given then by

$$B^T \tau = (\hat{z}, 0)^T$$

for $\tau \in \mathcal{M}^3$ and $z = (\hat{z}, \tilde{z})^T \in \mathbb{R}^N,\ \hat{z} \in \mathbb{R}^9,\ \tilde{z} \in \mathbb{R}^{N-9}$. Let $\mathfrak{sl}(3)$ be the set of all traceless 3×3-matrices, i.e.

$$\mathfrak{sl}(3) = \{v \in \mathcal{M}^3 \mid \operatorname{tr}\, v = 0\}.$$

Unknown in our small strain formulation are the displacement $u(x, t) \in \mathbb{R}^3$ of the material point x at time t and the vector of the internal variables $z = (p, \gamma) \in \mathbb{R}^{10}$. Here, $p(x, t) \in \mathfrak{sl}(3)$ denotes the non-symmetric infinitesimal plastic distortion and $\gamma(x, t) \in \mathbb{R}$ is the isotropic hardening variable with $p = Bz$. The condition $p(x, t) \in \mathfrak{sl}(3)$ expresses the plastic incompressibility.

Contrary to more classical strain gradient approaches, the models we study here feature a non-symmetric plastic distortion field $p \in \mathcal{M}^3$ (see [2,3]), a dislocation based energy storage based solely on $\|\mathrm{Curl} p\|$ and second gradients of the plastic distortion in the form of $\mathrm{Curl}\,\mathrm{Curl} p$ acting as dislocation based kinematical backstresses.

As is usual in plasticity theory, we split the total displacement gradient into non symmetric elastic and plastic distortions

$$\nabla u = e + p.$$

For invariance reasons (see [1,2] for more details), the elastic energy contribution may only depend on the elastic strains $\mathrm{sym}\, e = \mathrm{sym}(\nabla u - p)$.[3] While p is non-symmetric, a distinguishing feature of the model is that, similar to classical approaches, only the symmetric part $\varepsilon_p := \mathrm{sym}\, p$ of the plastic distortion appears in the local Cauchy stress σ, while the higher order stresses are non-symmetric (see [4,5] for more details).

The model equations of the problem are

$$-\mathrm{div}_x \sigma(x, t) = b(x, t), \tag{1}$$

$$\sigma(x, t) = \mathbb{C}[x](\mathrm{sym}(\nabla_x u(x, t) - Bz(x, t))), \tag{2}$$

$$\partial_t z(x, t) \in g\big(x, \Sigma^{\mathrm{lin}}(x, t)\big), \quad \Sigma^{\mathrm{lin}} = \Sigma^{\mathrm{lin}}_e + \Sigma^{\mathrm{lin}}_{\mathrm{sh}} + \Sigma^{\mathrm{lin}}_{\mathrm{curl}}, \tag{3}$$

$$\Sigma^{\mathrm{lin}}_{\mathrm{e}} = B^T \sigma, \qquad \Sigma^{\mathrm{lin}}_{\mathrm{sh}} = -Lz, \qquad \Sigma^{\mathrm{lin}}_{\mathrm{curl}} = -C_1 B^T \mathrm{Curl}\,\mathrm{Curl}(Bz),$$

which must be satisfied in $\Omega \times [0, T_e)$. Here, Σ^{lin} is the infinitesimal Eshelby stress tensor driving the evolution of the plastic distortion p. The initial condition and Dirichlet boundary condition are

$$z(x, 0) = 0, \quad x \in \Omega, \tag{4}$$

$$Bz(x, t) \times n(x) = 0, \quad (x, t) \in \partial\Omega \times [0, T_e), \tag{5}$$

$$u(x, t) = 0, \quad (x, t) \in \partial\Omega \times [0, T_e), \tag{6}$$

where n is a normal vector on the boundary $\partial\Omega$. For simplicity we consider only homogeneous boundary condition. The elasticity tensor $\mathbb{C}[x] : \mathcal{S}^3 \to \mathcal{S}^3$ is a linear, symmetric, uniformly positive definite mapping.

[3] Here, $\mathrm{sym} : \mathcal{M}^3 \to \mathcal{M}^3$ denotes the symmetrization operator.