Nonlinear Analysis: Real World Applications 25 (2015) 238–257

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Stability of a strong viscous contact discontinuity in a free boundary problem for compressible Navier–Stokes equations☆

Tingting Zheng

Computer and Message Science College, Fujian Agriculture and Forestry University, Fuzhou 350001, PR China

ARTICLE INFO

Article history:
Received 7 August 2014
Accepted 4 March 2015
Available online 16 May 2015

Keywords:
Free boundary problem
Navier–Stokes equations
Viscous contact discontinuity

ABSTRACT

In this study, we consider the nonlinear stability of a strong viscous contact discontinuity in a free boundary problem for the one-dimensional, full compressible Navier–Stokes equations in half space $[0, \infty)$. For the local stability of contact discontinuities, the local stability of a weak viscous contact discontinuity is well established, but for the global stability of an impermeable gas, fewer strong nonlinear wave stability results have been obtained, excluding zero dissipation or a $\gamma \to 1$ gas. Thus, our main aim is to determine the corresponding nonlinear stability result using the elementary energy method. For a certain class of large perturbation, we show that the global stability result can be obtained for a strong viscous contact discontinuity in Navier–Stokes equations.

1. Introduction

In this study, we consider a free boundary problem for a one-dimensional compressible viscous heat-conducting flow in the half space $\mathbb{R}_+ = [0, \infty)$, which is governed by the following initial–boundary value problem in Eulerian coordinates $(\tilde{x}, t)$:

$$\begin{cases} \tilde{\rho}_t + (\tilde{\rho}\tilde{u})_{\tilde{x}} = 0, \quad (\tilde{x}, t) \in \mathbb{R}_+ \times \mathbb{R}_+, \\ (\tilde{\rho}\tilde{u})_t + (\tilde{\rho}\tilde{u}^2 + \tilde{p})_{\tilde{x}} = \mu \tilde{u}_{\tilde{x}\tilde{x}}, \\ \left(\tilde{\rho}\left(\tilde{e} + \dfrac{\tilde{u}^2}{2}\right)\right)_t + \left(\tilde{\rho}\tilde{u}\left(\tilde{e} + \dfrac{\tilde{u}^2}{2}\right) + \tilde{p}\tilde{u}\right)_{\tilde{x}} = \kappa \tilde{\theta}_{\tilde{x}\tilde{x}} + (\mu \tilde{u}\tilde{u}_{\tilde{x}})_{\tilde{x}}, \end{cases} \tag{1.1}$$

where $\tilde{\rho}$, $\tilde{u}$, and $\tilde{\theta}$ are the density, velocity, and absolute temperature, respectively, $\mu > 0$ is the viscosity coefficient, and $\kappa > 0$ is the heat-conductivity coefficient. The pressure $p = \tilde{p}(\tilde{\rho}, \tilde{\theta})$ is related by the second law of thermodynamics. To simplify our problem, we focus on the perfect gas. In this situation,

$$\tilde{p}(\tilde{\rho}, \tilde{\theta}) = R\tilde{\theta}\tilde{\rho},$$

☆ This work was partially supported by National Natural Science Foundation of China (Grant No. 11426062). This work was partially supported by the Fujian Province Education Department Project (Grant No. JA12104).

E-mail address: asting16@sohu.com.

http://dx.doi.org/10.1016/j.nonrwa.2015.03.001
1468-1218/

$$\tilde{e}(\tilde{\rho},\tilde{\theta}) = \frac{R}{\gamma-1}\tilde{\theta} + \mathrm{const},$$

where $R > 0$ is the gas constant and $\gamma > 1$ is the adiabatic exponent. We consider the system (1.1) in the part $\tilde{x} > \tilde{x}(t)$, where $\tilde{x} = \tilde{x}(t)$ is a free boundary, with the following boundary condition

$$\frac{d\tilde{x}(t)}{dt} = \tilde{u}(\tilde{x}(t),t), \qquad \tilde{x}(0) = 0, \qquad \tilde{\theta}(\tilde{x}(t),t) = \theta_- > 0, \tag{1.2}$$

and

$$(\tilde{p} - \mu\tilde{u}_{\tilde{x}})|_{\tilde{x}=\tilde{x}(t)} = p_-, \tag{1.3}$$

which means that the gas is attached at the free boundary $\tilde{x} = \tilde{x}(t)$ to the atmosphere with pressure p_- and the initial data

$$(\tilde{\rho},\tilde{u},\tilde{\theta})(\tilde{x},0) = (\tilde{\rho}_0,\tilde{u}_0,\tilde{\theta}_0)(\tilde{x}), \qquad \lim_{\tilde{x}\to+\infty}(\tilde{\rho}_0,\tilde{u}_0,\tilde{\theta}_0)(\tilde{x}) = (\rho_+,0,\theta_+), \tag{1.4}$$

where ρ_+, θ_+ are positive constants and $\theta_0(0) = \theta_-$. We only consider the case of a single contact discontinuity, so we require

$$p_- = p_+ = R\theta_+\rho_+. \tag{1.5}$$

The boundary condition (1.3) means that the particles always stay on the free boundary $\tilde{x} = \tilde{x}(t)$, so if we use *Lagrangian* coordinates, then the free boundary becomes a fixed boundary, i.e.,

$$\begin{cases} v_t - u_x = 0, \quad (x,t) \in \mathbb{R}_+ \times \mathbb{R}_+, \\ u_t + \left(\dfrac{R\theta}{v}\right)_x = \mu\left(\dfrac{u_x}{v}\right)_x, \\ \dfrac{R}{\gamma-1}\theta_t + R\dfrac{\theta}{v}u_x = \kappa\left(\dfrac{\theta_x}{v}\right)_x + \mu\dfrac{u_x^2}{v}, \\ \theta|_{x=0} = \theta_-, \quad t > 0, \\ \left(\dfrac{R\theta_-}{v} - \mu\dfrac{u_x}{v}\right)(0,t) = p_+, \quad t > 0, \\ (v,u,\theta)|_{t=0} = (v_0,u_0,\theta_0) \to (v_+,0,\theta_+) \quad \text{as } x \to +\infty, \end{cases} \tag{1.6}$$

where v_+ and $\theta_\pm$ are given positive constants, and $v_0,\ \theta_0 > 0$. In fact, $v = 1/\rho(x,t),\ u = u(x,t),\ \theta = \theta(x,t)$, and $R\theta/v = p(v,\theta)$ are the specific volume, velocity, temperature, and pressure, respectively, as in (1.1).

In terms of various boundary values, Matsumura [1] classified all possible large-time behaviors of the solutions for one-dimensional (isentropic) compressible Navier–Stokes equations. In the case where $u(0,t) = 0$ (resp. $u(0,t) < 0$), the problem is called the *impermeable wall* (resp. *outflow*) problem when the boundary condition on density cannot be imposed. There have been many previous studies of the asymptotic behaviors of solutions to the initial–boundary value (or Cauchy) problem for the Navier–Stokes equations for these basic waves or their viscous versions (e.g., [2–8,1,9–23], and the references therein).

However, the problem of the stability of contact discontinuities is associated with linear degenerate fields and they are less stable than the nonlinear waves of the inviscid system (Euler equations). In [24,25], it was observed when the metastability of contact waves was studied for viscous conservation laws with artificial viscosity, then the contact discontinuity cannot be the asymptotic state for the viscous system and a diffusive wave, which approximates the contact discontinuity on any finite time interval, actually dominates the large-time behavior of solutions. The nonlinear stability of a contact discontinuity for the (full) compressible Navier–Stokes equations was then investigated in [2,5] for the free boundary value problem and [3,4] for the Cauchy problem.