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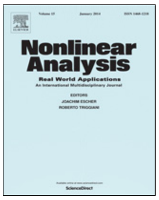

A PDE system modeling the competition and inhibition of harmful algae with seasonal variations

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ABSTRACT

In this paper, we study a reaction–diffusion–advection system modeling the competition of harmful algae with seasonal variations in a flowing water habitat. We assume that harmful algae produce toxins, which have inhibitory effects on their algal competitors, that is, the produced toxins can inhibit the growth of its competitor. For the single population model, we prove that the algae will be washed out eventually if the trivial periodic state is locally asymptotically stable, while there exists a unique positive periodic state which is globally attractive if the trivial periodic state is unstable. When there is mutual invasibility of both semitrivial periodic solutions of the two-species model, we are able to prove the existence of periodic coexistence state.

1. Introduction

Harmful algal blooms (HABs) have increased in frequency and intensity worldwide, in coastal and inland waters [1,2]. Associated implications include disruption of aquatic ecosystems and food webs, pollution of human food supplies and other health risks, and large fish kills. Those impacts have caused millions of dollars in economic losses. For example, *Prymnesium parvum* (golden algae) is a species particularly associated with numerous HABs in coastal and brackish inland waters. The toxins produced by *Prymnesium parvum* are lethal to fish and have toxic effects on many other aquatic organisms.

A potential technique has been suggested to manage and mitigate harmful algal blooms through flow manipulations in some riverine systems [3–5]. This possibility motivates researchers to do mathematical modeling describing the dynamics of harmful algal and the toxins produced in a flowing habitat. To understand the longitudinal distribution of algal abundance and toxicity arising along the axis of flow, the authors in [6] proposed two reaction–diffusion–advection systems modeling the growth of single population of harmful algae and the production and decay of its toxin in an idealized riverine reservoir, where a main channel was coupled to a hydraulic storage zone. We should point out that those PDE systems proposed

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in [6] only involve single population growth in spatially varying and temporally constant environments, and their mathematical analysis can be found in [7,8].

It was observed that harmful algae can produce toxins, which have inhibitory or allelopathic effects on their algal competitors [9–11]. The production of toxins can be an important factor that influences competitive dynamics [12]. Recently, the authors in [13,14] investigated mathematical models of two species competing for resources in a well-mixed chemostat, and species also compete through allelopathy. That is, each species produces a toxin that induces mortality in its competitor. We note that the authors in [15,16] also studied several mathematical models of the effect of inhibition on microbial competition in a chemostat environment. Another motivation of this paper is from [17], where a number of significant processes are incorporated, including production and degradation of toxins, allelopathy, zooplankton grazing and its inhibition by toxins produced by other organisms in a well-mixed habitat with temporal variations (see [17, Section 8]).

In this study, we aimed at understanding of competition between two algal species for a nutrient resource in a riverine habitat with spatiotemporal variations, and each species produces a toxin that reduces the growth rate of its competitor. Some potential complications are neglected here. For example, we remove the compartment of zooplankton from our system, and we ignore the hydraulic storage zone in our riverine reservoir. Rather than considering the habitat with a storage zone [18,17], our habitat is a riverine reservoir occupying a simple channel of longitudinally invariant cross-section that was formulated by Kung and Baltzis [19]. Such simplifications permit us to concentrate on the investigation of competition and inhibition between two independent species in spatially and temporally environments.

We first give some physical settings of the river system (see, e.g., [6,19,20]). Flow enters at the upstream end of the channel ($x = 0$), and an equal flow exits at the downstream end ($x = L$). Flow is parameterized at a constant dilution rate D (time^{-1}), and assuming constant water volume in the channel implies that advection occurs at a speed ν ($\nu = DL$). The flow of water in the channel in the direction of increasing x brings fresh nutrient at a concentration $R^{(0)}(t)$ into the reactor at $x = 0$ and carries medium, unused nutrient and organisms out of the reactor at $x = L$. Nutrient and organisms are assumed to diffuse throughout the vessel with the same diffusivity δ. Both advective and diffusive transports occur at the upstream boundary ($x = 0$). The downstream boundary is assumed to be a dam, over which there is advective flow but through which no diffusion can take place. Let $R(x,t)$ be the nutrient concentration at location x and time t; let $N_i(x,t)$ be the concentrations of species i in flowing habitats, respectively, and let $P_i(x,t)$ be the concentration of the inhibitor producing by $N_i(x,t)$. With these assumptions, we consider the following system:

$$\begin{cases} \dfrac{\partial R}{\partial t} = \delta\dfrac{\partial^2 R}{\partial x^2} - \nu\dfrac{\partial R}{\partial x} - q_{N1}f_1(R)N_1e^{-\mu_2P_2} - q_{N2}f_2(R)N_2e^{-\mu_1P_1}, & x\in(0,L),\ t>0,\\ \dfrac{\partial N_1}{\partial t} = \delta\dfrac{\partial^2 N_1}{\partial x^2} - \nu\dfrac{\partial N_1}{\partial x} + (1-k_1)f_1(R)N_1e^{-\mu_2P_2}, & x\in(0,L),\ t>0,\\ \dfrac{\partial N_2}{\partial t} = \delta\dfrac{\partial^2 N_2}{\partial x^2} - \nu\dfrac{\partial N_2}{\partial x} + (1-k_2)f_2(R)N_2e^{-\mu_1P_1}, & x\in(0,L),\ t>0,\\ \dfrac{\partial P_1}{\partial t} = \delta\dfrac{\partial^2 P_1}{\partial x^2} - \nu\dfrac{\partial P_1}{\partial x} + k_1f_1(R)N_1\dfrac{q_{N1}}{q_{P1}}e^{-\mu_2P_2}, & x\in(0,L),\ t>0,\\ \dfrac{\partial P_2}{\partial t} = \delta\dfrac{\partial^2 P_2}{\partial x^2} - \nu\dfrac{\partial P_2}{\partial x} + k_2f_2(R)N_2\dfrac{q_{N2}}{q_{P2}}e^{-\mu_1P_1}, & x\in(0,L),\ t>0, \end{cases} \tag{1}$$

with boundary conditions

$$\begin{cases} \nu R(0,t) - \delta\dfrac{\partial R}{\partial x}(0,t) = \nu R^{(0)}(t), \quad \dfrac{\partial R}{\partial x}(L,t) = 0,\\ \nu N_i(0,t) - \delta\dfrac{\partial N_i}{\partial x}(0,t) = \dfrac{\partial N_i}{\partial x}(L,t) = 0, \quad i=1,2,\\ \nu P_i(0,t) - \delta\dfrac{\partial P_i}{\partial x}(0,t) = \dfrac{\partial P_i}{\partial x}(L,t) = 0, \quad t>0, \end{cases} \tag{2}$$