Nonlinear Analysis: Real World Applications 26 (2015) 306–314

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

# Blow up of arbitrarily positive initial energy solutions for a viscoelastic wave equation[☆]

Haitao Song

*School of Mathematics and Statistics, Lanzhou University, Lanzhou, Gansu 730000, PR China*

A R T I C L E   I N F O

*Article history:*
Received 29 October 2014
Received in revised form 14 May 2015
Accepted 19 May 2015
Available online 6 July 2015

*Keywords:*
Blow-up
Finite time
Hyperbolic
Nonlinear damping
Viscoelastic

A B S T R A C T

In this paper, we consider the nonlinear viscoelastic equation:

$$u_{tt} - \triangle u + \int_0^t g(t-\tau) \triangle u(\tau) d\tau + |u_t|^{m-2} u_t$$
$$= |u|^{p-2} u, \quad \text{in } \Omega \times [0, T],$$

with initial conditions and Dirichlet boundary conditions. For nonincreasing positive functions g, we show the finite time blow up of some solutions whose initial data have arbitrarily high initial energy.



## 1. Introduction

In [1], Messaoudi considered the following initial–boundary value problem:

$$\begin{cases} u_{tt} - \triangle u + \displaystyle\int_0^t g(t-\tau)\triangle u(\tau)d\tau + a u_t |u_t|^{m-2} = b u|u|^{p-2}, & \text{in } \Omega \times (0,\infty) \\ u(x,t) = 0, \quad x \in \partial\Omega, \ t \geq 0, \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad x \in \Omega \end{cases} \tag{1.1}$$

where $\Omega$ is a bounded domain of $\mathbb{R}^n$ $(n \geq 1)$ with a smooth boundary $\partial\Omega, p > 2,\ m \geq 1, a, b > 0$, and $g : \mathbb{R}^+ \to \mathbb{R}^+$ is a positive nonincreasing function. He proved a blow up result for solution with negative initial energy if $p > m$, and a global result for $p \leq m$. This result was later improved by Messaoudi [2], to certain solutions with positive initial energy. For the problem (1.1) in $\mathbb{R}^n$ and with $m = 2$, Kafini and Messaoudi [3] showed, under suitable conditions on $g$ and initial data, that solution with negative energy

[☆] This work was partially supported by the Fundamental Research Funds for the Central Universities Grant (lzujbky-2010-68), and by China Postdoctoral Science Foundation funded project Grant (20100470882), and by Special Financial Grant from China Postdoctoral Science Foundation Grant (201104343), and by the Fundamental Research Funds for the Central Universities Grant (lzujbky-2013-10).

*E-mail address:* songhaitao@lzu.edu.cn.

http://dx.doi.org/10.1016/j.nonrwa.2015.05.015
1468-1218/

blows up in finite time. Song and Zhong [4] studied a nonlinear viscoelastic equation with strong damping:

$$\begin{cases} u_{tt} - \Delta u + \displaystyle\int_0^t g(t-\tau)\Delta u(\tau)d\tau - \triangle u_t = |u|^{p-2}u, & \text{in } \Omega \times [0,T], \\ u(x,t) = 0, \quad x \in \partial\Omega, \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \end{cases} \tag{1.2}$$

where $\Omega \subset \mathbb{R}^n$, is a bounded domain with a smooth boundary $\partial\Omega$. They proved a blow-up result for solutions with positive initial energy using the "potential well" theory introduced by Payne and Sattinger [5]. Very recently, Song and Xue [6] proved a blow up result for solution of Eq. (1.2) with arbitrarily high initial energy.

W. Liu [7] discussed the nonlinear viscoelastic problem

$$\begin{cases} |u_t|^\rho u_{tt} - \triangle u - \triangle u_{tt} + \displaystyle\int_0^t g(t-\tau)\triangle u(\tau)d\tau = bu|u|^{p-2}, & \text{in } \Omega \times (0,\infty) \\ u(x,t) = 0, \quad x \in \partial\Omega,\ t \geq 0, \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad x \in \Omega. \end{cases} \tag{1.3}$$

They proved the general decay result for the global solution and the finite time blow-up of solution.

In related work, Berrimi and Messaoudi [8] considered

$$u_{tt} - \triangle u + \int_0^t g(t-\tau)\triangle u(\tau)d\tau = u|u|^{p-2}, \quad \text{in } \Omega \times (0,\infty) \tag{1.4}$$

in a bounded domain and $p > 2$. They established a local existence result and showed that the local solution is global and decays uniformly if the initial data are small enough. In [9], the asymptotic stability and decay rates, for solutions of the wave equation in star-shaped domains, were established by combination of memory effect and damping mechanism. In [10], the existence and decay result for viscoelastic problems with nonlinear boundary damping have been proved. For further work on existence and decay of solutions of a viscoelastic equation, we refer to [11–24].

For the case when $g = 0$, it is worth mentioning some other papers in connection with existence, uniform decay and blow-up of solutions of nonlinear wave equations, e.g., [25–37] and references therein.

In this article, we consider the finite time blow up result for the viscoelastic wave equation (1.1) with initial data at high enough level. We will show, under suitable conditions on $g$, that there are solutions of (1.1) with arbitrarily high initial energy that blow up in finite time.

## 2. Blow up result

In the section, we shall discuss the blow up of problem (1.1). For simplicity, we assume $a = b = 1$. We first state a local existence theorem that can be established by combining arguments of [17,32].

**Theorem 2.1.** *Let $(u_0, u_1) \in H_0^1(\Omega) \times L^2(\Omega)$ be given. Let $g$ be a $C^1$ function satisfying*

$$1 - \int_0^\infty g(s)ds = l > 0. \tag{2.5}$$

*Let $m > 2$ and*

$$\begin{cases} 2 < p < \infty, \quad n = 1, 2, \\ 2 < p \leq \dfrac{2(n-1)}{n-2}, \quad n \geq 3. \end{cases} \tag{2.6}$$

*Then problem (1.1) has a unique local solution*

$$u \in C([0,T_m); H_0^1(\Omega)), \qquad u_t \in C([0,T_m); L^2(\Omega)) \bigcap L^2(\Omega \times [0,T_m)),$$

*for some $T_m > 0$.*