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Uniqueness and stability of positive steady state solutions for a ratio-dependent predator–prey system with a crowding term in the prey equation[☆]

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ABSTRACT

This paper deals with a ratio-dependent predator–prey system with a crowding term in the prey equation, where it is assumed that the coefficient of the functional response is less than the coefficient of the intrinsic growth rates of the prey species. We demonstrate some special behaviors of solutions to the system which the coexistence states of two species can be obtained when the crowding region in the prey equation only is designed suitably. Furthermore, we demonstrate that under some conditions, the positive steady state solution of the predator–prey system with a crowding term in the prey equation is unique and stable. Our result is different from those ones of the predator–prey systems without the crowding terms.

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1. Introduction

In the spatial population models, the coefficients often are assumed to be constants. To include spatial variations of the environment, some coefficients such as the growth rates, the crowding effects and the population interaction rates, should be replaced naturally by functions of the space variable x. For the class of population models in a spatially heterogeneous environment, it has been observed that in general, the behaviors of solutions to the class of population models are very sensitive to the change of certain coefficient functions in part of the underlying spatial region. These observations were successfully obtained by many pioneers such as Professors Brézis, Cirstea, Du, Fraile, García-Melián, López-Gómez and Ouyang *etc*. For example, for such a simple model $\partial_t u - \triangle u = \lambda u - a(x)u^2$ in Ω with the boundary condition $Bu = \alpha \frac{\partial u}{\partial \nu} + \beta u = 0$ which contains the following three cases $\alpha = 0$ and $\beta \neq 0$, or $\alpha \neq 0$ and $\beta = 0$, or $\alpha \neq 0$ and $\beta \neq 0$, where it is assumed that Ω_0 is an open and connected subset of Ω such that $a(x) = 0$ in Ω_0 and $a(x) > 0$ in $\Omega \backslash \overline{\Omega}_0$, H. Brézis and L. Oswald [1], Cantrell and Cosner [2], Cirstea and Radulescu [3–5], Du and Huang [6,7], J. M. Fraile and his coauthors [8], J. García-Melián and his coauthors [9,10], López-Gómez and his coauthors [11–18] and Ouyang [19] *etc.* obtained a critical patch size described by the principal eigenvalue $\lambda_1^D(\Omega_0)$, and demonstrated that if $\lambda \geq \lambda_1^D(\Omega_0)$, then $\lim_{t\to\infty} u(x,t) = \infty$ in $\overline{\Omega}_0$; while if $\lambda_1^B(\Omega) < \lambda < \lambda_1^D(\Omega_0)$, then $u(x,t)$ always is bounded in $\overline{\Omega}$ for all $t > 0$, and further

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its positive steady state solution is unique and globally stable. Meantime, López-Gómez and his coauthors [12–14,20,21] also studied other cases where Ω_0 is a multi-connected subset of Ω. By using the results above, [11–14,22–34] investigated some population models with the crowding effects or with protection zone, and obtained many interesting and valuable results with respect to the coexistence of two species. We find that these results of [7–14,20,22–34] are different from those ones of the corresponding systems in the spatially homogeneous environment. One can see [1–35] and references therein for a more detailed discussion on the models in a spatially heterogeneous environment.

In this paper, we will investigate the following predator–prey system which implies that the prey is with the crowding effect

$$\begin{cases} u_t - d_1\triangle u = \lambda u - a(x)u^2 - \dfrac{buv}{u+mv}, & x \in \Omega,\ t > 0, \\ v_t - d_2\triangle v = \mu v - v^2 + \dfrac{cuv}{u+mv}, & x \in \Omega,\ t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), & x \in \overline{\Omega}, \end{cases} \tag{1.1}$$

where $\lambda, \mu, b, c, m, d_1$ and d_2 are constants, and all of these constants are positive except μ which may take negative values; Ω is a bounded domain in $\mathbf{R}^N$ with smooth boundary $\partial\Omega$, $N \geq 1$, ν is the outward unit normal on $\partial\Omega$, and $\partial_\nu := \partial/\partial\nu$; $a(x)$ is a nonnegative continuous function in $\overline{\Omega}$. Moreover, there exists a region $\Omega_0 \subset \Omega$, such that $a(x) \equiv 0$ in $\overline{\Omega}_0$ and $a(x) > 0$ in $\overline{\Omega}\backslash\overline{\Omega}_0$. We assume that Ω_0 is an open and connected subset of Ω, and $\partial\Omega_0 \in C^2$. The homogeneous Neumann boundary conditions indicate that System (1.1) is self-contained with zero population flux across the boundary. For the meanings of other terms and coefficients in (1.1), one can refer to Refs. [36–40].

System (1.1) arises in mathematical biology as a ratio-dependent predator–prey model of two species which are interacting each other and migrating in the same habitat Ω, and System (1.1) is with the crowding effect in the prey equation. On the other hand, System (1.1) is also considered to be a control problem to the alien species u which is not the native species in Ω. The alien species u has not its preponderant native enemy in Ω, and most especially, the environment in Ω_0 is of benefit to the alien species u so that the amount of the species u in Ω_0 can become infinitely increasing. Hence, the ecological safety in Ω will be under threat.

As far as we are aware, when $a(x)$ is a positive constant, some authors investigated System (1.1). For example, [36–39] obtained the existence, stability and dynamical behaviors of positive solutions of (1.1). [40] investigated a ratio-dependent predator–prey system with cross-diffusion terms and demonstrated the existence of non-constant positive steady state solutions to the corresponding system with cross-diffusion terms.

In this paper, we will mainly investigate the impact of the region Ω_0 on the existence of the positive steady state solutions of (1.1). System (1.1) is singular and non-differentiable at the point $(u, v) = (0, 0)$, but we may re-define $\frac{buv}{u+mv} := 0$ at $(0, 0)$. Let $d_1 = d_2 = m = 1$. Then for $\lambda > b$, since every positive steady state solution $(u(x), v(x))$ of (1.1) satisfies that $u(x) > u_{\lambda-b}(x) > 0$ in $\overline{\Omega}$, any branch of the set of positive steady state solutions of (1.1) cannot be extended to $(\mu, 0, 0)$ for any μ, where $u_{\lambda-b}(x)$ is the unique positive solution of the equation $-\triangle u = (\lambda - b)u - a(x)u^2$ in Ω with the Neumann boundary condition when $0 < \lambda - b < \lambda_1^D(\Omega_0)$. Further, if $b < \lambda < \lambda_1^D(\Omega_0)$ and $\mu > -c$, then by using the local and global bifurcation theories, the fixed theory, the index theory and the stability theory, we will obtain the existence, uniqueness and stability of the positive steady state solution of (1.1); while for $\lambda > \lambda_1^D(\Omega_0)$ and $\mu > -c$, by constructing a suitable scalar equation and by using the comparison principle, we will obtain nonexistence of the positive steady state solution of (1.1). As for the dynamical behaviors of the positive solutions of (1.1) in various cases, we have also discussed it in [41], and obtained that under some conditions, the positive steady state solution of (1.1) is globally asymptotically stable provided $b < \lambda < \lambda_1^D(\Omega_0)$ and $\mu > -c$. For $0 < \lambda < b$, since the branch of the set of positive steady state solutions of (1.1) may be extended to $(\mu, 0, 0)$ for some μ, System (1.1) becomes more complicated. We will study the case $0 < \lambda < b$ in another paper.

This paper is organized as follows: in Section 2, we give some basic setups and preliminaries; in Section 3, we prove the existence, uniqueness and stability of positive steady state solution of (1.1) provided $b < \lambda < \lambda_1^D(\Omega_0)$ and $\mu > -c$; in Section 4, we prove nonexistence of the positive steady state solution of (1.1) provided $\lambda > \lambda_1^D(\Omega_0)$ and $\mu > -c$.

2. Preliminaries

In this section, we will give some basic setups and preliminaries as that in [26]. For the convenience of notation, we assume that $d_1 = d_2 = m = 1$. Thus, (1.1) becomes

$$\begin{cases} u_t - \triangle u = \lambda u - a(x)u^2 - \dfrac{buv}{u+v}, & x \in \Omega,\ t > 0, \\ v_t - \triangle v = \mu v - v^2 + \dfrac{cuv}{u+v}, & x \in \Omega,\ t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), & x \in \overline{\Omega}, \end{cases} \tag{2.1}$$

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