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A dynamic viscoelastic contact problem with normal compliance, finite penetration and nonmonotone slip rate dependent friction

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ABSTRACT

We consider a mathematical model which describes the dynamic evolution of a viscoelastic body in frictional contact with an obstacle. The contact is modelled with normal compliance and unilateral constraint, associated to a rate slip-dependent version of Coulomb's law of dry friction. In order to approximate the contact conditions, we consider a regularized problem wherein the contact is modelled by a standard normal compliance condition without finite penetrations. For each problem, we derive a variational formulation and an existence result of the weak solution of the regularized problem is obtained. Next, we prove the convergence of the weak solution of the regularized problem to the weak solution of the initial nonregularized problem. Then, we introduce a fully discrete approximation of the variational problem based on a finite element method and on a second order time integration scheme. The solution of the resulting nonsmooth and nonconvex frictional contact problems is presented, based on approximation by a sequence of nonsmooth convex programming problems. Finally, some numerical simulations are provided in order to illustrate both the behaviour of the solution related to the frictional contact conditions and the convergence result.

1. Introduction

Dynamic contact problems abound in industry and everyday life. For this reason, a considerable attention on modelling, mathematical analysis and numerical solution of such problems has been achieved recently in the engineering and mathematical literature. Owing to their inherent complexity, frictional contact phenomena lead to nonlinear, nonsmooth and nonconvex mathematical problems.

Various contact boundary conditions have been used to model contact phenomena, both in the engineering and mathematical topics and their modelling is still under investigation, see for instance [1–15] and the references therein. The establishment of improved frictional contact models is crucial to derive and analyse mathematical problems in good agreement with the physical point of view. Furthermore, this establishment permits to provide reliable numerical solution with accurate simulations.

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In the case of contact between a deformable body and a rigid foundation, Signorini condition [16] and normal compliance contact condition introduced in [17] are the most popular contact models used in the literature, see [4–6,10,11,14,15,18]. The Signorini condition describes the contact with a perfectly rigid foundation whereas the normal compliance describes the contact with an elastic foundation. These models are both questionable from a physical point of view since a perfectly rigid foundation is idealistic and an elastic foundation is too approximative and allows penetrations. A more realistic and general contact condition, called the normal compliance condition with unilateral constraint, was introduced in [19]. This model permits to control the penetration due to the normal compliance by the unilateral constraint; this penetration can be interpreted by the presence of microasperities on the surface of the foundation. This model contains as particular cases both the Signorini contact condition and the normal compliance condition, and it models the contact with an elastic–rigid foundation. Furthermore, the phenomenon of friction is inseparable from that of the contact. In particular, the interface asperities play a significant role in the establishment of the relationship between contact force and frictional force. The friction is generally modelled in the literature by the so-called Tresca and Coulomb friction laws. The classical monotone version of these laws with a friction coefficient assumed to be a constant, have shown their limits for friction-induced phenomena such as stick–slip motion. Then, many authors in the literature have introduced nonmonotone versions of friction laws [20–25]. For all these reasons, the consideration of a nonmonotone friction associated to a contact model based on normal compliance contact with unilateral constraint is of great interest.

The aim of this paper is to study a dynamic frictional contact problem in which the contact is modelled with normal compliance of such a type that the penetration, characterized by the size of the asperities, is restricted with unilateral constraint. Furthermore, the friction is modelled with a nonmonotone law in which the friction bound depends on the tangential velocities and the size of the asperities.

The analysis of mathematical models formulated by using a frictional contact condition with normal compliance, unilateral constraint and slip dependent friction can be found in [26,27]. There, the process considered was static and the material's behaviour was described with an elastic constitutive law. In [26], the friction bound is assumed to depend both on the tangential displacement and on the value of the depth of the penetration. The weak solvability of the model was proved by using arguments on pseudomonotone operators followed by a passage to the limit procedure; a convergence result was proved and its numerical validation was also provided. The work realized in [27] represents a continuation of [26] in which there is considered the different frictional contact model, able to describe the transition from the Coulomb to the Tresca friction laws during the passage from the normal compliance to unilateral constraint. Furthermore, in [27] the elasticity operator is nonlinear and the weak solvability of the problem was proved by using arguments of elliptic quasivariational inequalities; error estimates and numerical simulations are also provided.

The current paper represents a continuation of [26,27]. We consider a dynamic contact model with normal compliance, unilateral constraint and slip-dependent friction law for viscoelastic materials, and we provide its weak solvability and numerical solution together with simulations results. With respect to [26,27], the current paper presents some traits of novelties which we describe in what follows. The process is assumed to be dynamic and the material's behaviour is viscoelastic. Then, the frictional contact model we consider here is different, since the friction model based on that introduced in [27] uses a slip rate dependent friction. In the present paper we consider also two dynamic frictional contact problems with normal compliance, finite penetration and nonmonotone friction law. The first problem is characterized by normal compliance in which the penetration is restricted by unilateral constraint and the second problem represents a regularization of the first problem by considering unlimited penetration. For the regularized problem, we prove the existence of the weak solution. A trait of novelty of this paper arises from the fact that here we state and prove the convergence of the solution of the nonmonotone friction problem with normal compliance and unlimited penetration to the solution of the nonmonotone friction problem with normal compliance and finite penetration. This convergence result leads to the solvability of the initial nonregularized problem. The approach is based on [2,9], where a dynamic problem with Signorini condition and averaged contact force in the friction condition was considered, and the solution of the problem with infinite penetration was shown to converge in appropriate topology to the solution of the problem with unilateral constraints. Finally, we provide a numerical solution together with simulations which illustrate the mechanical behaviour of the frictional contact model and provide a numerical validation of the convergence result.

The rest of the paper is structured as follows. In Section 2 we introduce the notation we shall use as well as some preliminary material. In Section 3 we present the classical formulation of the frictional contact problem, we list the assumptions on the data and derive the variational formulation of the initial and approximate problems. Then, in Section 4 we state and prove the existence of the weak solution of the approximate problems. Section 5 is devoted to state and prove the convergence result. In Section 6 the numerical solution of the frictional contact problem is presented. And, finally, in Section 7 we present some numerical simulations on an academic two-dimensional example including a numerical validation of the convergence result.

2. Notation and preliminaries

We denote $\mathbb{R}_+ = [0, \infty)$. Moreover $d \in \{2, 3\}$ and $\Omega \subset \mathbb{R}^d$ is a bounded and open set with a sufficiently smooth (Lipschitz) boundary. The boundary of Ω is divided into three disjoint parts $\partial\Omega = \bar{\Gamma}_1 \cup \bar{\Gamma}_2 \cup \bar{\Gamma}_3$ that are such that Γ_1 and Γ_3 have positive boundary measure. The scalar product and norm in $\mathbb{R}^d$ are denoted by $\cdot$ and $\|\cdot\|$ respectively. By $\mathbb{S}^d$ we denote the space of symmetric $d \times d$ matrices and we use the notation $\boldsymbol{\sigma} : \boldsymbol{\varepsilon} = \sum_{i,j=1}^{d} \boldsymbol{\sigma}_{ij}\boldsymbol{\varepsilon}_{ij}$ valid for $\boldsymbol{\sigma}, \boldsymbol{\varepsilon} \in \mathbb{S}^d$.