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Approximate controllability for a class of hemivariational inequalities☆

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ABSTRACT

In this paper, we deal with the approximate controllability for control systems described by a class of hemivariational inequalities. Firstly, we introduce the concept of mild solutions for hemivariational inequalities. Then the approximate controllability is formulated and proved by utilizing a fixed-point theorem of multivalued maps and properties of generalized Clarke subdifferential.

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1. Introduction

The concept of hemivariational inequality was first introduced by Panagiotopoulos in 1981 and he applied the hemivariational inequalities to deal with the Mechanical problems with nonconvex and nonsmooth superpotentials in [1–3]. Currently, a growing number of scholars have made fruitful achievements on the existence of solutions for the hemivariational inequalities. For more details on these topics, one can see [4–12] and the references therein.

Recently, the study of the control problems for hemivariational inequality has become an active area of investigation by many researchers. In particular, Haslinger and Panagiotopoulos [13] showed the existence of optimal control pairs for a class of coercive hemivariational inequalities. In [14], Migórski and Ochal considered the optimal control problems for the parabolic hemivariational inequalities. J.Y. Park and S.H. Park [15,16] proved the existence of optimal control pairs to the hyperbolic systems. In [17,18], Tolstonogov paid his attention to the optimal control problems for subdifferential type differential inclusions.

Although the significant progresses have been made for the solvability and optimal control problems of hemivariational inequalities, it seems that there are still many unanswered questions and many interesting ideas in the making. It is well known that controllability plays a significant role in modern control theory and engineering since it is closely related to pole assignment, structural decomposition and quadratic optimal control. In recent years, there were a lot of excellent results on controllability problems (cf. [19–23]). However, to the best of our knowledge, the mathematical literature dedicated to the approximate controllability of the control systems described by the hemivariational inequalities is still untreated and

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this fact is the motivation of the present work. Our purpose of this paper is to provide some suitable sufficient conditions for the existence of mild solutions and approximate controllability of the following semilinear evolution hemivariational inequality:

$$\begin{cases} \langle -x'(t) + Ax(t) + Bu(t), v\rangle_H + F^0(t, x(t); v) \geq 0, \quad t \in J = [0, b], \ \forall v \in H, \\ x(0) = x_0, \end{cases} \tag{1.1}$$

where $\langle \cdot, \cdot\rangle_H$ denotes the scalar product of the separable Hilbert space H. $A : D(A) \subseteq H \to H$ is the infinitesimal generator of a C_0-semigroup $T(t)(t \geq 0)$ on H. The notation $F^0(t, \cdot; \cdot)$ stands for the generalized Clarke directional derivative (cf. [24]) of a locally Lipschitz function $F(t, \cdot) : H \to R$. The control function u takes value in $L^2(J, U)$ and the admissible controls set U is a Hilbert space, B is a bounded linear operator from U into H.

The paper is structured as follows. In Section 2, we will present some basic definitions and preliminary facts. In Section 3, some sufficient conditions are established for the existence of mild solutions of the system (1.1). In Section 4, the approximate controllability of the control system (1.1) is formulated and proved by applying a fixed point theorem. In Section 5, a concrete example is given to illustrate our main results.

2. Preliminaries

For a Banach space E with the norm $\|\cdot\|_E$, E^* denotes its dual and $\langle\cdot,\cdot\rangle$ the duality pairing of E^* and E. $C(J, E)$ denotes the Banach space of all continuous functions from $J = [0, b]$ into E with the norm $\|x\|_C = \sup_{t\in J} \|x(t)\|_E$.

Furthermore, given a Banach space E, we will use the following notations.

$\mathcal{P}_{f(c)}(E) := \{\Omega \subseteq E : \ \Omega \text{ is nonempty, closed (convex)}\}$;

$\mathcal{P}_{(w)k(c)}(E) := \{\Omega \subseteq E : \ \Omega \text{ is nonempty, (weakly) compact (convex)}\}$.

Next, we introduce some basic definitions and results from multivalued analysis. For more details on multivalued maps, please see the books [25,26]:

(i) For a given Banach space E, a multivalued map $F : E \to 2^E \setminus \{\emptyset\} := \mathcal{P}(E)$ is convex (closed) valued, if $F(x)$ is convex (closed) for all $x \in E$.

(ii) F is called upper semicontinuous (u.s.c. for short) on E, if for each $x \in E$, the set $F(x)$ is a nonempty, closed subset of E, and if for each open set V of E containing $F(x)$, there exists an open neighborhood N of x such that $F(N) \subseteq V$.

(iii) F is said to be completely continuous if $F(V)$ is relatively compact, for every bounded subset $V \subseteq E$.

(iv) Let (Ω, Σ) be a measurable space and (E, d) a separable metric space. A multivalued map $F : \Omega \to \mathcal{P}(E)$ is said to be measurable, if for every closed set $C \subseteq E$, we have $F^{-1}(C) = \{t \in \Omega : F(t) \cap C \neq \emptyset\} \in \Sigma$.

We shall make use of the following well-known results in this paper.

Theorem 2.1 (*Kuratowski–Ryll Nardzewski selection theorem, Theorem 2.2.1 of [26]*). *If (Ω, Σ) is a measurable space, X is a Polish space (i.e., separable completely metric space) and $F : \Omega \to \mathcal{P}_f(X)$ is measurable, then $F(\cdot)$ admits a measurable selection (i.e., there exists $f : \Omega \to X$ measurable such that for every $x \in \Omega$, $f(x) \in F(x)$).*

Lemma 2.2 (*Proposition 3.16 of [12]*). *Let (Ω, Σ, μ) be a σ-finite measure space, E be a Banach space and $1 \leq p < \infty$. If $f_n, f \in L^p(\Omega, E)$, $f_n \to f$ weakly in $L^p(\Omega, E)$ and $f_n(x) \in G(x)$ for μ-a.e. $x \in \Omega$ and all $n \in N$ where $G(x) \in \mathcal{P}_{wk}(E)$ for μ-a.e. $x \in \Omega$, then*

$$f(x) \in \overline{\text{conv}}\big(w\text{-}\limsup\{f_n(x)\}_{n\in N}\big) \quad \textit{for } \mu\textit{-a.e. on } x \in \Omega,$$

where $\overline{\text{conv}}$ denotes the closed convex hull of a set.

In what follows, let us proceed to the definition of the generalized gradient of Clarke for a locally Lipschitzian functional $h : E \to R$ (cf. [24]), we denote by $h^0(y; z)$ the Clarke generalized directional derivative of h at y in the direction z, that is

$$h^0(y; z) := \limsup_{\lambda\to 0^+,\ \xi\to y} \frac{h(\xi + \lambda z) - h(\xi)}{\lambda}.$$

Recall also that the generalized Clarke subdifferential of h at y, denote by $\partial h(y)$, is a subset of E^* given by

$$\partial h(y) := \{y^* \in E^* : h^0(y; z) \geq \langle y^*, z\rangle, \ \forall z \in E\}.$$

The following basic properties of the generalized directional derivative and the generalized gradient play important roles in our main results.

Lemma 2.3 (*Proposition 3.23 of [12]*). *If $h : \Omega \to R$ is a locally Lipschitz function on an open set Ω of E, then*

(i) *for every $z \in E$, one has $h^0(y; z) = \max\{\langle y^*, z\rangle : \text{ for all } y^* \in \partial h(y)\}$;*

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