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Thermoelastic rolling contact problems for multilayer materials

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ABSTRACT

The paper deals with the existence of solutions to the thermoelastic rolling contact problems for nonhomogeneous materials. One of the contacting surfaces is assumed to be covered with a graded material coating. The thermal and mechanical features of the coating material depend on its depth. The thermoelastic contact problem is governed by the system of mildly coupled evolutionary boundary value problems with discontinuous coefficients. Quasistatic approach is employed. This approach is based on the assumption that for the observer moving with the rolling body the displacement of the supporting foundation is independent on time. The Faedo–Galerkin approach combined with the penalization and smoothing approach are used to show the existence of solutions to this contact problem. The operator splitting method is used to solve the problem numerically. Numerical results indicating the reduction of mechanically and/or thermally induced stresses are provided.

1. Introduction

Contact problems involving transmission of loads across joints of surfaces covered with the functionally graded material coatings are of considerable research interest as well as practical importance in industry [1–4]. Functionally graded materials [4] are generally two-phase composites with continuously varying volume fractions. The results of experiments as well as numerical computations indicate [2,5,6] that these materials can reduce the magnitude of thermally and mechanically induced stresses and the likelihood of cracking due to excessive contact stress, suppress noise, increase the wear resistance and extend the structure service life. Coating materials have a broad range of applications [4] including such structures as bearings, gears, machine tools or abradable seals in gas turbines. The comprehensive review on the thermo-mechanical behavior, production techniques and potential main applications of functionally graded materials is presented in [4].

The thermoelastic contact problems involving homogeneous materials only have been considered by many authors. For details see the references in monographs [7–10]. Among others in [11] the existence and uniqueness of weak global solutions to the dynamic thermoviscoelastic contact problem is shown using a fixed point argument. The existence and uniqueness of solutions to thermoelastic contact is investigated in [12]. The contact behavior of functionally graded material structures has received increasing research effort in the recent years. Among others in [1–3,13,14] using the Fourier integral transform method these thermoelastic contact problems are formulated as Cauchy-type singular integral equations of the second type for the unknown contact pressure and solved numerically. In [15–17] the combined finite element and boundary element methods are used to solve numerically the transmission contact problems. The meshless method combined with radial basis functions is used in [18] to solve these problems numerically. Functionally graded material contact problems

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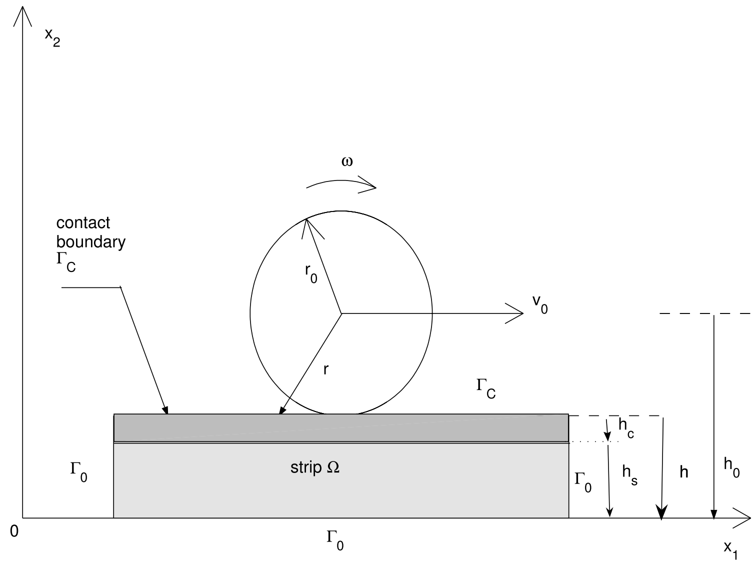

Fig. 1. Wheel rolling over the rail.

are also investigated as transmission problems. One-dimensional evolution transmission contact problems are considered in [19–21]. Quasistatic thermoelastic unilateral contact problem has been analyzed and numerically solved in [20] using the finite element approximation and the penalized formulation associated to the Signorini boundary condition. In [21] the existence of a weak solution is proved using the penalty approach and standard Galerkin approximation. Moreover it is shown that this solution decays exponentially as time goes to infinity. Evolution thermoelastic transmission problem in a radially symmetric domain is considered in [22] where the existence, uniqueness and regularity of solutions is shown as well as their exponential decay as time goes to infinity using the standard Galerkin approximation.

This paper is concerned with the rolling contact between a rigid wheel and an elastic rail lying on a rigid foundation. The contact and friction are governed by Signorini and Coulomb conditions, respectively [9,23]. The railhead where the wheel–rail contact occurs is assumed to be coated with the elastic graded material which properties depend on its depth according to the power law [2]. The displacement of the rail is governed by a hyperbolic boundary value problem. The heat flow generated due to friction is assumed to be transferred across the contact zone. This heat flow is governed by a parabolic conduction equation both inside the surface graded layer and inside the rail. The original contact phenomenon is governed by the coupled hyperbolic–parabolic boundary value system. Following [24] we take special features of this rolling contact problem and replace it by a quasistatic one. The proposed approach is based on the assumption that for the observer moving with the rolling wheel the displacement of the rail is independent of time. In this approach the inertial term of the displacement governing equation is replaced by the stationary term depending on wheel velocity and the derivative of displacement field with respect to spatial variables. This term is reflecting the dynamics of the body rather than completely neglected it as in the classical quasistatic formulation [8,25]. The existence of solutions to this contact problem is shown. The problem has been numerically solved using the finite element method as a discretization method and an operator splitting method [5]. Assuming the rail is covered with an elastic coating the contact pressure and the temperature distributions in the wheel–rail contact zone are computed. The numerical results are provided and discussed.

2. Problem formulation

Consider a rigid wheel rolling over an elastic rail occupying a bounded domain $\Omega \subset R^2$ and lying on a rigid foundation (see Fig. 1). Domain Ω is assumed to consists from two elastic and thermally conductive layers Ω_c and Ω_s such that $\bar{\Omega} = \bar{\Omega}_c \cup \bar{\Omega}_s$. The material features of the coating layer Ω_c are described by a continuous function dependent on spatial variables. In numerical computations this function is approximated by a piecewise constant function.

The material parameters of the substrate layer Ω_s are assumed to be constant. These layers have thicknesses h_c and h_s, respectively. Moreover $h_c < h_s$ and $h = h_c + h_s$ denotes the thickness of the strip Ω. By Γ_c and Γ_s we denote the boundaries of Ω_c and Ω_s respectively. The boundary Γ of the strip Ω is assumed to consist from two mutually disjoint parts Γ_0 and Γ_C such that $\Gamma = \bar{\Gamma}_0 \cup \bar{\Gamma}_C$ and $meas(\Gamma_0) > 0$. Obviously $\Gamma = \bar{\Gamma}_c \cup \bar{\Gamma}_s \setminus (\Gamma_c \cap \Gamma_s)$. A wheel rolls along the upper surface Γ_C of the strip. The wheel has radius r_0, rotating speed ω and linear velocity $V = v_0$. The axis of the wheel is moving along a straight line at a constant altitude h_0 where $h_0 < h + r_0$, i.e., the wheel is pressed in the elastic strip. There is no mass forces in the strip. The head and tail ends of the strip are clamped along the boundary Γ_0, i.e., we assume that the length of the strip is much bigger than the radius of the wheel. The contact and friction conditions are prescribed on a portion Γ_C of the boundary Γ. The frictional sliding causes heat generation as well as heat flow through the contact zone.