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Stress-driven local-solution approach to quasistatic brittle delamination

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ABSTRACT

A unilateral contact problem between elastic bodies at small strains glued by a brittle adhesive is addressed in the quasistatic rate-independent setting. The delamination process is modeled as governed by stresses rather than by energies. This leads to a specific scaling of an approximating elastic adhesive contact problem, discretized by a semi-implicit scheme and regularized by a BV-type gradient term. An analytical zero-dimensional example motivates the model and a specific local-solution concept. Two-dimensional numerical simulations performed on an engineering benchmark problem of debonding a fiber in an elastic matrix further illustrate the validity of the model, convergence, and algorithmical efficiency even for very rigid adhesives with high elastic moduli.

1. Introduction

Fracture mechanics and mathematical theory of rate-independent processes have achieved considerable progress during the past decades. A number of models have been developed in engineering and in mathematics accounting for different features of the materials. Even more, particular models admit various concepts of solutions which, in combination with the specific model, can describe certain specific aspects of the process under consideration. However, it is well recognized that solutions to rate-independent systems governed by non-convex potentials, as it is the case in fracture models, may exhibit sudden jumps, i.e., sudden rupture. Various concepts of weak solutions – in this rate-independent case also called *local solutions* – have been devised, ranging from *energetic solutions*, which *conserve energy*, to approximable, vanishing-viscosity, BV-, ε-sliding, or maximally-dissipative solutions, cf. in particular [1–7]. Let us emphasize that the adjective "local" does not refer to local existence in time — just opposite, the local solutions here will always exist globally in time and in qualified cases will coincide with conventional weak solutions. Independently of [7], local solutions have been invented also in [8] under the name "maximally-dissipative minimizers". In convex situations, all these concepts essentially coincide with each other but in general nonconvex situations they are very different. This is related with the conceptual question whether rather energy or rather stress governs the inelastic process in question and it also has to do with the issue of global versus local minimization, cf. the discussion in mathematical literature [9,8] and in engineering [10], and also the examples

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[11, Sect. 9], [12, Sect. 6], or [13, Example 7.1]. In particular, *energetic solutions*, which form a sub-class of the *local solutions*, are known to exhibit a tendency to early jumps that may not be physically meaningful. Therefore, in this article, we will focus to another type of local solutions.

We will restrict the fracture process to a prescribed interface, and thus focus ourselves on a so-called *delamination* problem, called also an adhesive contact problem, cf. [14,15] for a survey of various models. In this context, it was already observed in [16] that the local solutions obtained by semi-implicit time discretization nicely coincide numerically with the vanishing-viscosity solutions in all investigated examples; of course, energy conservation is lost for such local but non-energetic solutions. Mathematical justification of such, essentially, *stress-driven evolution* has been scrutinized in [5] for the setting of delamination with *adhesive contact*, where it reveals a certain connection with the maximal-dissipation principle, and then numerically in [17], for a slightly more general adhesive model. One of the motivations for the stress-driven local solutions is to avoid the undesired delamination due to big stored energy in a large bulk even under small stress — also called a "long-bar paradox" in [2], cf. also [5,6].

In view of their good performance in the setting of adhesive contact, cf. [17,16], it is the aim of this paper to establish the notion of stress-driven local solutions also for the setting of brittle delamination. For this, we will valorize the method of an *adhesive contact approximation of brittle delamination* studied in [18] in the context of energetic solutions. More specifically, in this paper we address a *delamination* problem of two *elastic bodies at small strains* glued along a contact boundary $\Gamma_{\rm C}$ by a *brittle* adhesive with a prescribed fracture toughness. The interface $\Gamma_{\rm C}$ separates the body located in the domain $\Omega \subset \mathbb{R}^d$, with $2 \le d \in \mathbb{N}$, into two parts, Ω_- and Ω_+. In the spirit of Generalized Standard Materials, in particular Frémond's concept of adhesion [19], the degradation state of the adhesive during the time span $[0,T]$ is captured by an additional internal variable $z : [0,T] \times \Gamma_{\rm C} \to [0,1]$, where $z(t,x) = 1$ stands for the fully intact state, whereas $z(t,x) = 0$ models complete rupture (=debonding) of the adhesive in the material point $x \in \Gamma_{\rm C}$ at time $t \in [0,T]$. This approach essentially admits arbitrarily shaped $(d-1)$-dimensional crack area evolving along the interface $\Gamma_{\rm C}$. Moreover it allows to display both adhesive contact and brittle delamination in a unified way: The brittle delamination model describes the crack growth in a brittle adhesive. Expressed in terms of the displacement field $u : [0,T] \times (\Omega \setminus \Gamma_{\rm C}) \to \mathbb{R}^d$ and the delamination variable $z : [0,T] \times \Gamma_{\rm C} \to [0,1]$, the assumption is that the displacements cannot jump on $\operatorname{supp} z(t) \subset \overline{\Gamma}_{\rm C}$, the spatial support of z at time t, while on the crack set $\Gamma_{\rm C} \setminus \operatorname{supp} z(t)$ they may jump. This so-called *brittle constraint* can be expressed with the aid of the indicator function

$$J_\infty([\![u]\!], z) := \begin{cases} 0 & \text{if } \big|[\![u]\!]\big| = 0 \text{ in } x \in \operatorname{supp} z, \\ \infty & \text{otherwise.} \end{cases} \tag{1.1}$$

In the adhesive contact model, due to the more viscous properties of the adhesive, the two parts of the body can be slightly detached from each other without the adhesive degradation. In other words, here, the displacements u are allowed to jump on $\operatorname{supp} z$ at a current time, but the jump is penalized by the *adhesive contact term*

$$J_k([\![u]\!], z) = \frac{k}{2} z \big|[\![u]\!]\big|^2. \tag{1.2}$$

Thus, (1.2) can be used to relax the non-convex and nonsmooth constraint (1.1). The contact between the two components of the body will be considered *unilateral* but frictionless, which is encoded in the *non-penetration condition*

$$I_{\rm C}(x, [\![u]\!]) := \begin{cases} 0 & \text{if } [\![u]\!] \cdot \mathrm{n}(x) \ge 0, \\ \infty & \text{otherwise,} \end{cases} \tag{1.3}$$

where $\mathrm{n}(x)$ denotes the unit normal vector pointing from Ω_- to Ω_+ at $x \in \Gamma_{\rm C}$. Any rate effects, such as viscosity, inertia or temperature dependence, are neglected and the problem is thus completely *rate-independent*. The time-continuous brittle problem, involving (1.1), will be approximated by adhesive contact problems, involving (1.2) with $k \to \infty$, and discretized in time by a semi-implicit scheme scaled in such a way that *stress-driven nucleation of the crack* will be correctly modeled in the limit. As a side effect, an efficient robust numerical strategy will be devised. The convergence proof will require a BV-type gradient regularizing term scaled to zero in the limit model. However, by compactness, the BV-property of the approximating solutions is passed on to the (approximable) solutions of the limit model. It can be understood in a similar way that also the information on the stress-driven nature of the delamination process is handed down from the approximating time-discrete adhesive problems to the time-continuous brittle limit.

The scaling used here in the context of stress-driven local solutions, cf. (2.11) and (3.1c) below, differs significantly from the scaling applied in [18] for the setting of energetic solutions. At first glance, the new scaling looks rather surprising because asymptotically the fracture toughness tends to 0, and thus, the dissipated energy due to delamination vanishes. But this scaling has already been investigated numerically in engineering literature for static problems close to the onset of rupture, cf. [20, Formula (16)] or [21, Formula (7)]. It is recognized that this scaling has the capacity to predict correctly crack nucleations. On the other hand, due to the typical stress concentration on the crack tips of already existing cracks, this scaling usually leads to the effect of too easily propagating cracks (i.e. propagating already under small driving stress and nearly not dissipating any energy) and therefore this simple model must be combined with some plasticity mechanism (usually called a ductile fracture, cf. e.g. [22–25]), which is however far beyond of the scope of this paper.

The plan of the paper is the following: In Section 2, we introduce the problem, the notion of local solution, and motivate the new scaling for the adhesive models towards the brittle limit on a simple explicit example. Then, in Section 3, we