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Dynamical analysis of a model of harmful algae in flowing habitats with variable rates

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ABSTRACT

This study considers an advection–dispersion–reaction model of harmful algae in a flowing water habitat. The habitat is selected such that the main channel is coupled with a storage zone. The rates are considered to be time dependent instead of constant, which is more natural. The introduction of variable rates makes the system difficult to study. We prove the well-posedness and boundedness using the evolution semigroup approach. Often, it is difficult to find an analytical solution for a given model, thus approximate solutions are very useful and it is important to study them. In the second part, we use a discretization method to find the approximate solution of our model system.

1. Introduction

Ecologists have observed that plankton do not develop in small, low-order streams with fast currents. In this ecosystem, the suspended algae arise transiently from the bottom. The development of plankton is more likely to occur in broad rivers and riverine reservoirs compared with small streams. At this point, streams are warmer and supplied less abundantly with leaves compared with upstream sections. These larger streams remain well oxygenated because air is entrained by the turbulent flow in riffles. Light can reach the river benthos in shallow water, which increases in-stream primary productivity. The study of models of flowing water habitat is very important from an ecological viewpoint. In a flowing water habitat, the flow enters one boundary of a channel and then exits at another. This process supplies nutrients, and then removes nutrients and organisms. Both advection and diffusion transport organisms and nutrients to the habitat domain [1–6]. For this type of system, many basic questions are still difficult to answer such as persistence, mainly because the rapid advection prevents persistence. This matter was discussed eloquently by Hsu et al. in [1]. In another study by Grover et al. [2], it was noted that the presence of hydraulic storage zones in flowing water habitats is a possible solution to this persistence paradox. It has been observed that advective and diffusive transport occur only in the flowing zone but not in the storage zone, although the nutrient concentration and population densities vary with location in both the flowing channel and the storage zone. When modeling the system of riverine reservoirs and fluvial lakes formed in drowned river valleys, which have strong advective flows, the flow reactor and its modifications are very important [2]. An idealized riverine reservoir is a system where a main channel with advective transport and dispersion is coupled to a hydraulic storage zone, which is represented by an ensemble of fringing coves on the shoreline. Grover et al. [2], considered these systems to study the distributions of algal abundance and toxicity under various flows. For more studies of this topic interested readers can refer to [1] and the references therein.

In the present study, we consider a one-dimensional model with a simple habitat geometry and transport processes with variable rates. Our main aim is to study spatial variations in harmful algae, as well as their toxin production and decay in

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riverine reservoirs. Many previous studies have discussed the autonomous model, for example [1,2]. In the present study, we adopt a continuum approach using an advection–dispersion–reaction system to resolve the transport and biochemical reaction kinetics along the main channel of a riverine reservoir. In order to simplify the model and to concentrate on the study of population dynamics and toxin production and decay, we also neglect several potential complications, including vertical stratification, light limitation, and higher trophic levels, according to Hsu et al. [1]. The model still supports elaboration using the details necessary to model specific flowing water systems in Texas, where harmful blooms have occurred. Based on the argument outlined above, it has been suggested that this type of model system could be used to model other riverine system worldwide by selecting suitable conditions and parameters [2]. There have been many excellent studies of this topic, especially by Hsu et al. and Grover et al. [1–3], which interested readers may explore.

The remainder of this paper is organized as follows. In Section 1, we define an evolution semigroup and discuss the existence and uniqueness of its solution. In Section 2, we analyze the qualitative behaviors of the solutions. In Section 3, we present the discretization scheme for the model system, which can be implemented to obtain numerical solutions. Finally, a brief conclusion is given.

2. Preliminaries

Motivated by the previous research mentioned in the introduction section, the present study addresses the existence, uniqueness, stability, and approximations of the solutions for the following system of advection–diffusion equations,

$$
\begin{aligned}
\frac{\partial R(t,x)}{\partial t} &= \delta(t)\frac{\partial^2 R(t,x)}{\partial x^2} - v(t)\frac{\partial R(t,x)}{\partial x} - q_N(t)(f(R(t,x)) - m(t))N(t,x) + \alpha(t)(R_S(t,x) - R(t,x)),\\
\frac{\partial N(t,x)}{\partial t} &= \delta(t)\frac{\partial^2 N(t,x)}{\partial x^2} - v(t)\frac{\partial N(t,x)}{\partial x} - \alpha(t)(N_S(t,x) - N(t,x)) + (f(R(t,x)) - m(t))N(t,x),\\
\frac{\partial C(t,x)}{\partial t} &= \delta(t)\frac{\partial^2 C(t,x)}{\partial x^2} - v(t)\frac{\partial C(t,x)}{\partial x} + \epsilon p(R(t,x), N(t,x)) + \alpha(t)(C_S(t,x) - C(t,x)) - k(t)C(t,x),\\
\frac{\partial R_S(t,x)}{\partial t} &= -\alpha(t)\frac{A}{A_S}(R_S(t,x) - R(t,x)) - q_N(t)(f(R_S(t,x)) - m(t))N_S(t,x),\\
\frac{\partial N_S(t,x)}{\partial t} &= -\alpha(t)\frac{A}{A_S}(N_S(t,x) - N(t,x)) + (f(R_S(t,x)) - m(t))N_S(t,x),\\
\frac{\partial C_S(t,x)}{\partial t} &= -\alpha(t)\frac{A}{A_S}(C_S(t,x) - C(t,x)) + \epsilon p(R_S(t,x), N_S(t,x)) - kC_S(t,x),
\end{aligned}
\tag{2.1}
$$

for $(t,x) \in \mathbb{R}^+ \times (0,L)$, with the boundary conditions

$$
\begin{aligned}
&-\delta(t)\frac{\partial R(t,0)}{\partial x} + v(t)R(t,0) = v(t)R_0,\\
&-\delta(t)\frac{\partial N(t,0)}{\partial x} + v(t)N(t,0) = 0\\
&-\delta(t)\frac{\partial C(t,0)}{\partial x} + v(t)C(t,0) = 0,\\
&\frac{\partial R(t,L)}{\partial x} = \frac{\partial N(t,L)}{\partial x} = \frac{\partial C(t,L)}{\partial x} = 0,
\end{aligned}
\tag{2.2}
$$

and the initial conditions

$$
\begin{aligned}
&R(0,x) = R_0(x) \geq 0, \qquad N(0,x) = N_0(x) \geq 0, \qquad C(0,x) = C_0(x) \geq 0,\\
&R_S(0,x) = R_{0S}(x) \geq 0, \qquad N_S(0,x) = N_{0S} \geq 0, \qquad C_S(0,x) = C_{0S}(x) \geq 0,
\end{aligned}
\tag{2.3}
$$

for $x \in (0,L)$.

The functions $R(t,x)$, $N(t,x)$, and $C(t,x)$ denote the dissolved nutrient concentration, algal abundance, and dissolved toxin concentration at time t and location x in the flowing channel, respectively. In addition, the functions $R_S(t,x)$, $N_S(t,x)$, and $C_S(t,x)$ denote the dissolved nutrient concentration, algal abundance, and dissolved toxin concentration at time t and location x in the storage zone, respectively. For simplicity, we assume that the nutrient content of the algae that die is recycled instantaneously and locally. Moreover, the degradation of toxins is assumed to follow first order kinetics with a decay coefficient k, which we assume is a function of time t.

The mortality of algae is assumed to be the variable $m(t)$. We also assume that all of the coefficients are continuous functions of time, which are bounded below and above by some positive constants, where this condition is a natural assumption. The restrictions are given as:

$$
\delta_* \leq \delta(t) \leq \delta^*, \qquad v_* \leq v(t) \leq v^*, \qquad q_{N*} \leq q_N(t) \leq q_N^*, \qquad \alpha_* \leq \alpha(t) \leq \alpha^*, \qquad m_* \leq m(t) \leq m^*. \tag{2.4}
$$