



Structural stability for a thermal convection model with temperature-dependent solubility



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ABSTRACT

We study a problem involving thermosolutal convection in a fluid when the solute concentration is subject to a chemical reaction in which the solubility of the dissolved component is a function of temperature. When the spatial domain is a bounded one in \mathbb{R}^2 we show that the solution depends continuously on the reaction rate using true *a priori* bounds for the solution when the chemical equilibrium function is an arbitrary function of temperature.

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1. Introduction

The problem of double diffusion convection in a horizontal layer of fluid with simultaneous chemical reaction is well studied, cf. the account in Straughan [1], pp. 225–237. Indeed the governing equations for such a chemical reaction problem are derived by Morro and Straughan [2], employing ideas of continuum thermodynamics. It is also worth drawing attention to the fact that similar equations arise in fluid phase change problems where continuum thermodynamic theories are again employed and analysed by Berti et al. [3], Berti et al. [4], Bonetti et al. [5], Bonetti et al. [6], Fabrizio [7] and Fabrizio et al. [8–10]. A particular reaction where the solute concentration is subject to a chemical reaction in which the solubility of the dissolved component is a linear function of the temperature has been analysed recently in a porous medium context by Pritchard and Richardson [11], by Wang and Tan [12], and by Malashetty and Biradar [13]. The object of this paper is to analyse the effect of such a class of reaction terms in a fluid. The dependence on temperature is here taken to be arbitrary and not only linear. We allow the fluid to occupy a bounded domain $\Omega \subset \mathbb{R}^2$ with boundary Γ sufficiently smooth to allow application of the divergence theorem. We are particularly interested to investigate the continuous dependence of the solution on the reaction rate. It is very reasonable to expect that in such a mathematical model one can achieve an appropriate type of continuous dependence and this kind of stability problem belongs to the important class of structural stability questions.

Structural stability, or continuous dependence on the model itself, is a concept at least as important as the classical idea of stability which involves continuous dependence on the initial data, as explained in some detail by Hirsch and Smale [14]. Structural stability was studied in elasticity by Knops and Payne [15], then further advanced in a variety of continuum mechanical contexts by Payne [16–18] and by Knops and Payne [19]. Since then structural stability studies in continuum mechanics have proved very popular as witnessed by the works of Aulisa et al. [20,21], Celebi et al. [22], Chirita et al. [23],

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Ciarletta et al. [24,25], Hoang and Ibragimov [26,27], Hoang et al. [28], Kalantarov and Zelik [29], Kanded [30], Kang and Park [31], Kelliher et al. [32], Li et al. [33], Liu [34,35], Liu et al. [36,37], Ouyang and Yang [38], Passarella et al. [39], Ugurlu [40], You [41].

2. Fundamental equations

The fundamental model we study is based upon the equations of balance of momentum, balance of mass, conservation of energy, and conservation of salt concentration, adopting a Boussinesq approximation in the body force term in the momentum equation. Thus, let $v_i(\mathbf{x}, t)$, $p(\mathbf{x}, t)$, $T(\mathbf{x}, t)$ and $C(\mathbf{x}, t)$ denote velocity, pressure, temperature and salt concentration, where $\mathbf{x} \in \Omega$, t denote time, with $0 < t < \mathcal{T}$, for some $\mathcal{T} < \infty$. Then, the equations of momentum, mass, energy, and salt concentration are taken to be

$$\begin{aligned} \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} &= -\frac{\partial p}{\partial x_i} + \Delta v_i + g_i T - h_i C, \\ \frac{\partial v_i}{\partial x_i} &= 0, \\ \frac{\partial T}{\partial t} + v_i \frac{\partial T}{\partial x_i} &= \Delta T, \\ a \frac{\partial C}{\partial t} + b v_i \frac{\partial C}{\partial x_i} &= \Delta C + Lf(T) - KC, \end{aligned} \quad (1)$$

where g_i , h_i represent gravity vectors and without loss of generality we assume that

$$|\mathbf{g}|, |\mathbf{h}| \leq 1.$$

Standard indicial notation is used throughout with a repeated index denoting summation over 1 and 2, and a , b , L and K are positive constants. Eqs. (1) follow in practice by employing a Boussinesq approximation which accounts for variable C allowing the incompressibility condition to hold, cf. Fife [42], pp. 72–74.

The function f is at least C^1 and the term $Lf(T)$ is analogous to the chemical equilibrium term, C_{eq} , in Pritchard and Richardson [11], Wang and Tan [12], and Malashetty and Biradar [13], although all of these writers assume $C_{eq}(T)$ is a linear function of T . The terms $Lf(T) - KC$ in equation (1)₄ correspond to the mass supply term m_α Morro and Straughan in [2]. The justification for this is as given by Pritchard and Richardson in [11] who write $Lf(T) - KC = K(C_{eq} - C)$, C_{eq} being a chemical equilibrium term. The logic is that in chemical equilibrium the chemical reaction arises solely due to the term $K(C_{eq}(T) - C)$.

In general (1)₄ holds with $a = b = 1$. However, we allow for different a and b since these coefficients will change under a rescaling, i.e. under a different non-dimensionalization.

Eqs. (1) hold in the domain $\Omega \times (0, \mathcal{T})$, together with the initial conditions,

$$\begin{aligned} v_i(\mathbf{x}, 0) &= v_i^0(\mathbf{x}), \\ T(\mathbf{x}, 0) &= T_0(\mathbf{x}), \\ C(\mathbf{x}, 0) &= C_0(\mathbf{x}), \end{aligned} \quad (2)$$

for $\mathbf{x} \in \Omega$, and the boundary conditions

$$\begin{aligned} v_i(\mathbf{x}, t) &= 0, \\ T(\mathbf{x}, t) &= g(\mathbf{x}, t), \\ C(\mathbf{x}, t) &= h(\mathbf{x}, t), \end{aligned} \quad (3)$$

$\mathbf{x} \in \Gamma$, $t \in [0, \mathcal{T})$.

Since we are interested in studying continuous dependence on the reaction rates L and K , we let (u_i, p_1, T_1, C_1) and (v_i, p_2, T_2, C_2) be two solutions to (1)–(3) for the same initial and boundary data, but for different reaction coefficients (L_1, K_1) and (L_2, K_2) . To progress we now introduce the difference variables (w_i, π, θ, ϕ) and l and k by

$$\begin{aligned} w_i &= u_i - v_i, & \pi &= p_1 - p_2, & \theta &= T_1 - T_2, \\ \phi &= C_1 - C_2, & l &= L_1 - L_2, & k &= K_1 - K_2. \end{aligned} \quad (4)$$

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