Nonlinear Analysis: Real World Applications 22 (2015) 54–65

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Analysis of spatiotemporal patterns in a single species reaction–diffusion model with spatiotemporal delay[☆]

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HIGHLIGHTS

- The conditions of both spot and stripe patterns are given.
- The multiple scale method is used to obtain the amplitude equations.
- The patterns are plotted to show the physical implication.

ARTICLE INFO

Article history:
Received 7 March 2014
Accepted 28 July 2014
Available online 25 August 2014

Keywords:
Spatiotemporal patterns
Turing bifurcation
Spatiotemporal delay
Multiple scale method
Amplitude equations

ABSTRACT

Employing the theories of Turing bifurcation in the partial differential equations, we investigate the dynamical behavior of a single species reaction–diffusion model with spatiotemporal delay. The linear stability and the conditions for the occurrence of Turing bifurcation in this model are obtained. Moreover, the amplitude equations which represent different spatiotemporal patterns are also obtained near the Turing bifurcation point by using multiple scale method. In Turing space, it is found that the spatiotemporal distributions of the density of this researched species have spots pattern and stripes pattern. Finally, some numerical simulations corresponding to the different spatiotemporal patterns are given to verify our theoretical analysis.

1. Introduction

In the past decades, spatiotemporal patterns, especially in biology, chemistry and physics, have been investigated by using reaction–diffusion equations [1–3]. Currently, the spatiotemporal patterns of population distribution have been a hot topic in ecology because species abundance changes not only in time but also in space. As known, these patterns present the spatial heterogeneity of population distribution and are similar with many field observations. In order to study the spatiotemporal distribution of population, a lot of mathematical models have been proposed on the basis of all kinds of factors including noise [4,5], delay [6,7], spatiotemporal delay [8,9] and so on. Employing these mathematical models, many researchers have paid their great attention to dynamical behavior and bifurcation phenomena in the reaction–diffusion equations to explore the mechanisms of spatiotemporal patterns formation. From the point of view of pattern formation, these spatiotemporal patterns can be classified by three different bifurcation mechanisms: (i) oscillatory in time but uniform in space induced by spatial homogeneous Hopf bifurcation [10–17], (ii) periodic in space but stationary in time induced by Turing bifurcation [18–20], (iii) periodic in space and oscillatory in time induced by wave bifurcation [21,22]. However

☆ This research is supported by the State Key Program of National Natural Science Foundation of China under Grant No. 11032009, and the National Natural Science Foundation of China under Grant No. 11272236.

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http://dx.doi.org/10.1016/j.nonrwa.2014.07.013
1468-1218/

the previous results about the spatiotemporal pattern phenomena in the biological reaction–diffusion system were mainly discussed in two species models such as predator–prey system, competition system, corporation system and so on. There are few relevant results about a single species model with spatiotemporal delay induced by aggregation effects. Spatiotemporal patterns in such a single species model with spatiotemporal delay thus need to be performed and this is the aim of our present work.

Spatiotemporal delay, also called nonlocal effects in space and time, was firstly proposed by N.F. Britton in 1990. This was based on the assumption that the intra-specific competition at a point depended on not the point but the neighborhood of this point in the past time when the aggregation effects were considered. As N.F. Britton pointed out that the spatiotemporal delay played an important role in the pattern formation [9]. However, this model was not investigated thoroughly by N.F. Britton. To illustrate, he did not consider spots pattern and stripes pattern. In order to explore the occurrence of these spatiotemporal patterns, we will employ the multiple scale method to solve this problem.

Multiple scale method has been widely used both in the ordinary differential equations and in the partial differential equations. This is a kind of perturbation method, extensively used in nonlinear dynamics. In particular, this method is very practical in dealing with dynamical and engineering problems which are described by the ordinary differential equations [23,24], the delayed differential equations [25,26], and the partial differential equations [3], respectively.

To explore the effects of time delay on the spatiotemporal pattern formation, the rest of the paper is organized as follows. In Section 2, the model with spatiotemporal delay is introduced and linear stability analysis of this model is carried out. As a result, the critical values for Hopf bifurcation and Turing bifurcation are given respectively. Section 3 employs the multiple scale method about Turing bifurcation to give the amplitude equations of Turing patterns. In Section 4, the stability analysis of the amplitude equations is performed, and the conditions for Turing bifurcation for the different spatiotemporal patterns, such as stripes pattern and spots pattern, are further investigated. To verify our theoretical analysis, some numerical simulations corresponding to the different Turing patterns are also given. Finally, some conclusions are drawn in Section 5.

2. Model and bifurcation analysis

In this section, we mainly pay our attention to the single species reaction–diffusion model with spatiotemporal delay, which was firstly proposed by N.F. Britton [8,9] with the following form,

$$u_t = d\Delta u + ru(1 + \alpha u - \beta u^2 - (1 + \alpha - \beta)(f * *u)), \tag{2.1}$$

where

$$(f * *u)(x, t) = \int_{-\infty}^{t} \int_{\mathbb{R}^2} f(x - y, t - s)u(y, s)dyds, \quad \text{and} \quad f(x, t) = \frac{1}{4\pi t} e^{-\frac{|x|^2}{4t}} \frac{1}{\tau} e^{-\frac{t}{\tau}},$$

for $(x, t) \in \mathbb{R}^2 \times (-\infty, \infty)$. The parameters $r, \alpha, \beta, d, \tau$ are positive constants, and $\Delta = \frac{\partial^2}{\partial X^2 + \partial Y^2}$ is Laplacian operator in two dimensional space. And we further assume $1 + \alpha - \beta > 0$. So, model (2.1) can be regarded as with effects of diffusion, aggregation, reproduction and competition for space and resources under these assumptions. The terms in model (2.1) have the following biological interpretation according to [9] respectively: the term $\alpha u (\alpha > 0)$, is a measure of the advantage to individuals in aggregating or grouping; the term $-\beta u^2$ in model (2.1) denotes competition for space (rather than resources), which impedes population growth and stops the population density from ever exceeding a certain value; finally, the integral term of model (2.1) reflects competition between the individuals for food resources. In this model, the spatiotemporal convolution models a distributed temporal delay in the dynamics and the spatial component arises from the fact that individuals take time to move. The previous researches demonstrated that there were three kinds of bifurcation solutions: (i) steady spatially periodic structure solutions, (ii) periodic standing wave solutions, and (iii) periodic traveling wave solutions when he only considered the nonlocal effect in space. These bifurcation solutions would lead to the formation of spatiotemporal patterns, including uniform temporal oscillations, stationary spatially periodic patterns, standing waves and wave trains in [9]. These similar problems were also pointed out in [27,28].

In order to explore the other spatiotemporal patterns of system (2.1), we employ the Turing bifurcation theories to accomplish it. For the simplicities of our research, we let $v(x, t) = (f * *u)(x, t)$, which leads to system (2.1) be transformed into a two dimension partial differential system as follows:

$$\begin{cases} u_t = d\Delta u + f_1(u, v), \\ v_t = \Delta v + f_2(u, v), \end{cases} \tag{2.2}$$

where $f_1(u, v) = ru(1 + \alpha u - \beta u^2 - (1 + \alpha - \beta)v), f_2(u, v) = \frac{1}{\tau}(u - v)$. System (2.2) of reaction–diffusion equations is the starting point for much of the analysis in this paper, and has three equilibria $(0, 0)$, $(-\frac{1}{\beta}, -\frac{1}{\beta})$, and $(1, 1)$. We will mainly analyze system (2.2) to get the dynamical behavior of system (2.1). So, from the biological point of view, we are mainly interested in the third equilibrium point since this point corresponds to the maximum state of population.

Now, employing the linear analysis of system (2.2) near the equilibrium point $(1, 1)$, we can get linear equations as follows:

$$\begin{cases} u_t = d\Delta u + a_{11}u + a_{12}v, \\ v_t = \Delta v + a_{21}u + a_{22}v, \end{cases} \tag{2.3}$$