Nonlinear Analysis: Real World Applications 22 (2015) 66–83

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Dynamics in a diffusive plankton system with delay and toxic substances effect☆

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HIGHLIGHTS

- The dynamics of a reaction–diffusion plankton system with delay and toxic substances effect is considered.
- Existence and priori bound of a solution for a model without delay are shown.
- The global asymptotic stability of the axial equilibrium is obtained.
- The stability of the positive equilibrium and the existence of Hopf bifurcation are obtained.
- The direction of the Hopf bifurcation and the stability of the bifurcating periodic solutions are determined.

ARTICLE INFO

Article history:
Received 27 March 2014
Received in revised form 16 July 2014
Accepted 19 July 2014
Available online 25 August 2014

Keywords:
Reaction–diffusion
Stability
Hopf bifurcation
Delay
Plankton

ABSTRACT

The dynamics of a reaction–diffusion plankton system with delay and toxic substances effect is considered. Existence and priori bound of a solution for a model without delay are shown. Global asymptotic stability of the axial equilibrium is obtained. The stability of the positive equilibrium and the existence of Hopf bifurcation are investigated by analyzing the distribution of eigenvalues. And the properties of Hopf bifurcation are determined by the normal form theory and the center manifold reduction for partial functional differential equations. Some numerical simulations are carried out for illustrating the theoretical results.

1. Introduction

Plankton is one of the important elements of the marine ecosystem. Phytoplankton not only plays an important role in the primary production but also absorb not less than half of the carbon dioxide which may be contributing to global warming [1]. The importance of plankton for ecosystems is nowadays widely recognized [2].

There are many scholars who attempted to propose differential equations as plankton models to describe the dynamics of plankton ecosystem. In order to simulate the effect of nutrients on the growth of planktonic communities, Beretta et al. [3] constructed a chemostat-type nutrient–plankton model. They assumed that the limiting nutrient is partially recycled after the death of the organisms. It has been observed that plankton models with delayed nutrient recycling exhibit

☆ This research is supported by National Natural Science Foundation of China (Nos. 11031002, 11371111), Research Fund for the Doctoral Program of Higher Education of China (No. 20122302110044), Natural Science Foundation of Heilongjiang Province of China (No. A201422) and Program for Young Teachers Scientific Research in Qiqihar University (No. 2012k-M28).

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http://dx.doi.org/10.1016/j.nonrwa.2014.07.010
1468-1218/

very interesting and rich dynamics (see [3–5]). Some scholars considered the effect of light and nutrient on the growth of phytoplankton, and discussed the dynamics of this type model (see [6–10]).

In recent years, there has been a global increase in harmful plankton blooms [2,11,12]. There are several papers which attempt to construct different mathematical models to describe the reduction of zooplankton due to toxin production phytoplankton (see [13,14,12]). Chattopadhyay et al. [13] discussed an ordinary differential equation model on harmful algal blooms. Chattopadhyay and Sarkar [14] considered a differential equation plankton model with discrete delay in the following form:

$$\begin{cases} \dfrac{dP}{dt} = rP\left(1-\dfrac{P}{K}\right) - \alpha PZ, \\ \dfrac{dZ}{dt} = \beta PZ - \mu Z - \dfrac{\theta P(t-\tau)Z}{\gamma + P(t-\tau)}, \end{cases} \tag{1.1}$$

where P and Z represent the density of phytoplankton and zooplankton population, respectively, $\alpha(>0)$ is the specific predation rate and $\beta(>0)$ represents the ratio of biomass consumed per zooplankton for the production of new zooplankton, $\mu(>0)$ is the mortality rate of zooplankton, $\theta(>0)$ is the rate of toxin production per phytoplankton species, $\gamma(>0)$ is the half saturation constant and $\tau(\geq 0)$ is the discrete time delay.

In the lakes or oceans, plankton may move for many reasons, such as currents and turbulent diffusion. So in more realistic ecological models, the diffusion should be considered. Assume that the water is closed, with no plankton species entering and leaving the water at the boundary. We consider spatial changes in both the species, and a spatial model analogue of the model (1.1) presented takes the following form:

$$\begin{cases} \dfrac{\partial P(x,t)}{\partial t} = d_1 \Delta P(x,t) + rP(x,t)\left(1-\dfrac{P(x,t)}{K}\right) - \alpha P(x,t)Z(x,t), \\ \dfrac{\partial Z(x,t)}{\partial t} = d_2 \Delta Z(x,t) + \beta P(x,t)Z(x,t) - \mu Z(x,t) - \dfrac{\theta P(x,t-\tau)Z(x,t)}{\gamma + P(x,t-\tau)}, \quad x \in \Omega,\ t>0, \\ \dfrac{\partial P(x,t)}{\partial \nu} = 0, \qquad \dfrac{\partial Z(x,t)}{\partial \nu} = 0, \quad x \in \partial\Omega,\ t>0, \\ P(x,t) = \Phi_1(x,t) \geq 0, \qquad Z(x,t) = \Phi_2(x,t) \geq 0, \quad x\in\Omega,\ -\tau \leq t \leq 0, \end{cases} \tag{1.2}$$

where $d_1, d_2 > 0$ denote the diffusion coefficients of phytoplankton and zooplankton, respectively, ν is the outward unit normal vector on $\partial\Omega$.

In the rest of this paper, we will assume, unless we state explicitly otherwise, that $\Omega = (0, l\pi),\ l > 0$, where $l\pi$ denotes the depth of the water column. In this case, the homogeneous Neumann boundary condition means that no plankton species is entering or leaving the column at the top or the bottom.

Define the real-valued Sobolev space

$$X := \{(u, v) \in H^2(0, l\pi) \times H^2(0, l\pi) | u_x = v_x = 0, x = 0, l\pi\},$$

with inner product $\langle\cdot,\cdot\rangle$.

System (1.2) always has the following nonnegative equilibria: a trivial equilibrium $E_0(0,0)$, an axial equilibrium $E_1(K,0)$.

We make the following assumption

$$\text{(H1)} \quad \theta < (\beta K - \mu)\left(1 + \frac{\gamma}{K}\right).$$

Then, under (H1), system (1.2) has a positive equilibrium denoted by $E_*(P_*, Z_*)$, where

$$P_* = \frac{-(\beta\gamma - \mu - \theta) + \sqrt{(\beta\gamma - \mu - \theta)^2 + 4\beta\gamma\mu}}{2\beta},$$
$$Z_* = \frac{r}{\alpha}\left(1 - \frac{P_*}{K}\right).$$

The rest of the paper is organized as follows. In Section 2, the existence of the solution to the diffusion system without delay is proved, and a priori bound of the solution is also established. In Section 3, by using the upper and lower method, we show the global asymptotic stability of the axial equilibrium. In Section 4, by analyzing the distribution of the roots of the characteristic equation, the stability of the positive constant steady state and the existence of Hopf bifurcation are obtained. In Section 5, by applying the normal form theory and the center manifold reduction of partial functional differential equations, an explicit algorithm for determining the direction of the Hopf bifurcation and the stability of the bifurcating periodic solutions is derived. Finally, some numerical simulations are presented to illustrate the theoretical results.