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Analysis and simulations of multifrequency induction hardening[☆]

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ABSTRACT

We study a model for induction hardening of steel. The related differential system consists of a time domain vector potential formulation of Maxwell's equations coupled with an internal energy balance and an ODE for the volume fraction of *austenite*, the high temperature phase in steel. We first solve the initial boundary value problem associated by means of a Schauder fixed point argument coupled with suitable a priori estimates and regularity results. Moreover, we prove a stability estimate entailing, in particular, uniqueness of solutions for our Cauchy problem. We conclude with some finite element simulations for the coupled system.

1. Introduction

In induction hardening a coil that is connected to an alternating current source generates a periodically changing electro-magnetic field. The temporal changing magnetic flux induces a current in the workpiece that is enclosed by the induction coil. Due to the resistance of the workpiece, some part of the power is transformed into eddy current losses that result in Joule heating. The latter leads to a change of microstructure to the high temperature phase in steel called austenite. After switching off the current and possibly a short holding time the workpiece is cooled rapidly and the austenite layer produced upon heating is transformed into another phase called martensite, responsible for the desired hardening effect.

Induction heat treatments can easily be integrated into a process chain. Moreover, they are energy efficient since the heat is generated directly in the workpiece. That is why induction hardening is still the most important surface treatment technology.

Due to the skin effect, the eddy currents tend to distribute in a small surface layer. The penetration depth of these eddy currents depends on the material and essentially on the frequency. Therefore, it is difficult to obtain a uniform contour hardened zone for complex workpiece geometries such as gears using a current with only one frequency. If for example, a high frequency (HF) is applied, then the penetration depth is small and it is possible to harden only the tip of the gear. With a medium frequency (MF) it is possible to heat the root of the gear, but not the tip. With a single frequency, a hardening of

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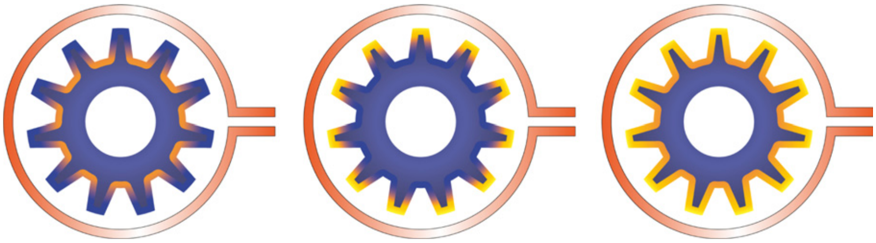

Fig. 1. The effect of medium-, high- and multifrequency induction heating. MF (left): only the root of the gear is heated, HF (middle): only the tip of the gear is heated, MF + HF (right): tip and root of the gear are heated.

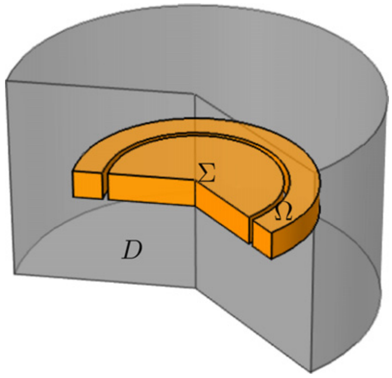

Fig. 2. Domain D consisting of the inductor Ω, the workpiece Σ and the surrounding air.

the complete tooth can only be achieved by increasing the heating time. But then, the complete tooth is heated beyond the austenitization temperature, which results in a complete martensitic structure of the tooth after quenching, which is not desirable, since this will foster fatigue effects.

Recently, a new approach has been developed which amounts to supplying medium and high frequency powers simultaneously on the induction coil. This concept is called *multifrequency induction hardening*, see also Fig. 1.

The inductor current consists of a medium frequency fundamental oscillation superimposed by a high frequency oscillation. The amplitudes of both frequencies are independently controllable, which allows separate regulation of the respective shares of the output power of both frequencies according to the requirements of the workpiece. This provides the ability to control the depth of hardening at the root and the tip of the tooth individually [1].

The main building blocks for a mathematical model of multifrequency hardening are an eddy current formulation of Maxwell's equations in the time domain, coupled to the balance of internal energy to describe the temperature evolution, and a model for the evolution of the high temperature phase austenite. Since the electric conductivity and the magnetic permeability may depend on temperature and/or the phase fractions and the Joule effect in the time domain is modelled by the square of the time derivative of the magnetic vector potential we are faced with a strongly coupled nonlinear system of evolution equations.

In two recent papers the simpler frequency domain situation of Joule heating has been studied. In [2] the Boccardo–Gallouët approach has been used to prove existence of a weak solution, while in [3] new regularity results established in [4] have been used to prove existence and stability in the frequency domain setting. In [5,6] the eddy current model has been considered in the time domain. In the former the existence of a weak solution to the fully coupled model has been proven, in the latter also stability results are established, but only for a model with one-sided coupling in which the electric conductivity and permeability are assumed to be constant.

The main novelty of the present paper is an existence and stability result for the strongly coupled time domain eddy current and Joule heating system. The paper is organized as follows. In the next section we derive the model equations and show that it complies with the second law of thermodynamics in form of the Clausius–Duhem inequality. In Section 3 we formulate the main mathematical results, the proofs of which are given in Section 4. The last section is devoted to presenting some results of numerical simulations for the coupled system based on a two time step approach with sequential decoupling of the evolution equations.

2. The model

We restrict to the following idealized geometric setting (cf. Fig. 2). Let $D \subset \mathbb{R}^3$ be a domain containing the inductor coil Ω and the workpiece Σ. Assume that $\overline{\Omega} \subset D$, $\overline{\Sigma} \subset D$, $\overline{\Omega} \cap \overline{\Sigma} = \emptyset$ and $\partial\Omega$, $\partial\Sigma$, ∂D are of class $C^{1,1}$. Call $G = \Omega \cup \Sigma$ the set of conductors and define the space–time domain as $Q = \Sigma \times (0, T)$.