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Existence and sharp decay rate estimates for a von Karman system with long memory

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ABSTRACT

A nonlinear model described by von Karman equations with long memory is considered. Hadamard wellposedness of weak solutions, regularity of solutions and intrinsic decay rate estimates for the energy are established by assuming that the memory kernel g satisfies the inequality introduced in Alabau-Boussouira and Cannarsa (2009): $g' \leq -H(g)$, where $H(s)$ is a given continuous, positive, increasing, and convex function such that $H(0) = 0$. The decay rates obtained are optimal in the sense that they reconstruct decay rates assumed on relaxation kernel. The novelty of the paper is at the level of both: the results obtained and the methodology applied.

1. Introduction

1.1. Description of the problem

This paper is concerned with the existence and uniform decay rates of the energy for solutions of the following viscoelastic model of scalar von Karman equations:

$$\begin{cases} \partial_t^2 u + \Delta^2 u - [u, v] - g * \Delta^2 u = 0 \quad \text{in } \Omega \times (0, \infty), \\ \Delta^2 v + [u, u] = 0 \quad \text{in } \Omega \times (0, \infty), \\ u = v = 0 \quad \text{on } \Gamma \times (0, \infty), \\ \partial_\nu u = \partial_\nu v = 0 \quad \text{on } \Gamma \times (0, \infty), \\ u(x, 0) = u^0(x); \qquad \partial_t u(x, 0) = u^1(x) \quad \text{in } \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^2$ with smooth boundary Γ and

$$(g * z)(t) = \int_0^t g(t - \tau) z(\tau)\, d\tau, \tag{1.2}$$

$$[u, v] = u_{xx} v_{yy} + u_{yy} v_{xx} - 2u_{xy} v_{xy}. \tag{1.3}$$

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Scalar von Karman models describe vertical oscillations of nonlinear plates subject to *large* displacements. It is essential to note that the model introduced above does not account for regularizing effects of rotational inertia which make the nonlinear term subcritical (see [1] for the discussion). Analysis of von Karman equations has acquired considerable attention in the mathematical literature. Both wellposedness and long time behavior of solutions have been prime objects of the studies. Starting with existence of weak solutions obtained by Faedo Galerkin methods [2,3], study of regularity of solutions [4,5] and of long time behavior subject to frictional or thermal damping [5,6] were natural topics for consideration. A longstanding open problem of *uniqueness of weak solutions* has been positively settled in [7]. This gave foundations to the study of stabilization problems with boundary frictional damping or thermoelastic damping obtained in [8–14]. Exhaustive list of references can be found in a recent monograph [1]. For reader's orientation we shall also mention several references pertaining to *full von Karman systems*, where plane accelerations are accounted for. Hadamard well-posedness of weak solutions has been proved in [15,16] while uniform stability with boundary feedback has been considered in [17,18,15,19].

In the context of *viscoelastic plates with memory terms*, we refer to [20–24]. In [20] it was shown that solutions to viscoelastic plate with *additional frictional damping* decay to zero as time goes to infinity. In [21–24] models accounting for regularizing effects of inertial terms were considered. For these dynamics von Karman nonlinearity is subcritical—thus the issue of wellposedness and regularity of solutions is standard from the point of view of modern nonlinear theory. The main results obtained in these works are decay rates for the viscoelastic energy of weak and strong solutions which reflect the decay rates of relaxation kernels. Both exponential and polynomial cases were studied. For other treatments of viscoelastic models we refer to [25–30]. While these papers do not deal specifically with nonlinear plate theory, treatment of viscoelasticity at more abstract level has had useful consequences in the developments of plate theory.

In discussing past literature relevant to a more general study of viscoelasticity one should start with the pioneering contributions [31,32] which treat the issues related to the wellposedness and asymptotic behavior of viscoelastic solutions. No decay rate estimates were exhibited in these works. Subsequently, a nice monograph on viscoelasticity [33] presents various developments in this area. However, first results pertaining to decay rate estimates for solutions of viscoelastic models are [34] (see also [24] for nonlinear models). This area of research was followed with a considerably large number of papers published on viscoelasticity and their asymptotic behavior quantized by decay rates obtained for the total energy: [25–28, 35–37,29,30,38,39]. More recently some (not so many) results related on locally distributed viscoelastic effects, (where the concept of propagation of the damping becomes critical) can be found in [40–42] and the most recent work in this direction [43].

In the present paper viscoelastic scalar von Karman model is considered *without* accounting for regularizing effects of rotational inertia. The main goal of the manuscript is to provide

- wellposedness of both weak and regular solutions,
- an unified theory of decay rates of the energy associated with *very general* viscoelastic kernels characterized by nonlinear differential inequalities.

Then our goal is to show that the decay rates for the energy function are optimal in the sense that they decay qualitatively the same as the viscoelastic kernels do.

In order to accomplish this goal, we shall pursue the strategy introduced in [44], which is based on characterizing decay rates via a suitably constructed ODE. This will lead to the results which are both *sharp and general*. It will be shown that the decay rates of the energy corresponding to relaxation kernel g are driven by an *ODE*, which is quantitatively the same as the one describing transient behavior of the relaxation function. As a by-product we shall obtain full ranges of parameters for polynomially decaying kernels whereas previous literatures required stringent assumption imposed on the ranges of the parameters.

There are four main ingredients in the approach developed for the solution to the problem stated above: (i) *extensive use of "sharp regularity"* of Airy's stress function [7], (ii) *construction and use of multipliers* dealing with viscoelasticity and introduced originally in [27] and later used in [28,38,45] and references therein; (iii) *construction of ODE type comparison theory* which is rooted in convex analysis. The proposed method is based on an adaptation to viscoelasticity of "ODE" technique developed for frictional damping in [44]; (iv) *construction of suitable "control functions"*, which in the absence of quantitative information on relaxation kernels lead to sharp, precise characterization of the decay rates. This step has been introduced in [45] and fully developed and optimized in [46].

Well-posedness of finite energy solutions will be shown by resorting to "sharp regularity" of Airy's stress functions used in the context of viscoelastic dynamics. Sharp regularity is critical not only for Hadamard wellposedness of weak solution and for a construction of strong solutions, but also for energy estimates yielding the decays of energy. The latter are necessary for rigorous justification of PDE estimates. The multipliers, inspired by works of [45,27], allow to reduce the analysis to final length time segments rather than resorting to Lyapunov function type of arguments used in all prior literature. Convexity based approach allows to characterize decay rates via suitably constructed ODE. In order to establish sharp and general uniform decay rates we employ the method recently developed in [46] and also [45]. Sharp decay rates for an abstract linear viscoelastic model with the relaxation kernel satisfying *equality* $g' + H(g) = 0$ were announced (with a short outline of the proof) in [26]. Indeed, it is claimed in [26] that the energy of a linear viscoelastic model decays at the same rate as the kernel $g(s)$. In a more general case of *inequality* $g' + H(g) \leq 0$, uniform decay rates without precise quantification are also claimed in [26].