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Optimization of species survival for logistic models with non-local dispersal☆

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ABSTRACT

To describe the effects of spatial dispersal strategies and heterogeneous environments on the dynamics of species, we study a logistic model with non-local dispersals for single species. The aim of this paper is to study the optimization of species survival and the investigations are carried out from two aspects: optimization of survival chances and maximization of total population of species. W.l.o.g., assume that the spatial distribution of resources $m \in L^\infty(\Omega)$ satisfies

$$-1 \le m(x) \le 1 \quad \text{and} \quad \frac{1}{|\Omega|}\int_\Omega m(x)dx \le \alpha, \quad \alpha \in (-1, 1).$$

Our main results indicate that $m = \chi_E - \chi_{\Omega\setminus E}$, where $E \subset \Omega$ is measurable with $|E|/|\Omega| = (1+\alpha)/2$ and χ_E is the characteristic function of E, is optimal for both the survival chances and total population of species respectively.

1. Introduction

One of the major problems in mathematical ecology is to understand the effects of spatial dispersal strategies and heterogeneous environments on the distribution of species. In recent years, non-local dispersal, which describes the movement of organisms between non-adjacent spatial locations, has attracted more and more attention in ecology. We refer [1–16] and references therein for more details.

In particular, in [1,13], the authors restrict their attention to logistic models with the simplified non-local operator $\mathcal{L}u = d(\bar{u} - u)$, where the symbol $\bar{f}$ means the integral average of f, that is, $\bar{f} := \frac{1}{|\Omega|}\int_\Omega f dx$. See [13] for the derivation and interpretation of this operator. From the viewpoint of biology, this simplified non-local dispersal operator corresponds to the case that the movement distance of the species is much larger than the diameter of the habitat. In [1,13], the dynamics of single species and multiple species models in heterogeneous environments with this non-local operator has been thoroughly studied.

Motivated by the work [1,13], the aim in this paper is to further investigate how spatial variation in the environment of the habitat affects the maintenance and total population of species using the following single species model with non-local diffusion:

$$u_t = d[\bar{u} - u] + u[m(x) - u] \quad \text{in } \Omega \times (0, \infty). \tag{1.1}$$

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In this model, $u(t, x)$ denotes the population densities of the species, $d > 0$ is the dispersal rate, which measures the total number of dispersal individuals per unit time, and the function $m(x)$ represents its intrinsic growth rate.

For convenience, we set up the following assumptions first. Throughout this paper, we always assume that

(A) $m(x) \in L^\infty(\Omega)$ and it is non-constant.

Then let $\alpha \in (-1, 1)$ and assume that

(A1) $-1 \le m(x) \le 1$ and $\bar{m} \le \alpha$.

From the viewpoint of biology, two problems will be studied in this paper:

(P1) What is the optimal spatial arrangement of the favorable and unfavorable parts of the habitat for species to survive?
(P2) What is the optimal spatial distribution of resources for which the total population of species can be maximized?

These two problems can be rephrased as mathematical questions respectively as follows:

Question 1.1. *Among all functions $m(x)$ that satisfy* (A) *and* (A1), *which $m(x)$ will yield the largest range of d for which* (1.1) *admits a positive steady state?*

Question 1.2. *Among all functions $m(x)$ that satisfy* (A) *and* (A1), *which $m(x)$ will yield the largest $\int_\Omega \theta dx$, where θ denotes the positive steady state of* (1.1) *whenever it exists?*

Before proceeding with the discussions, we prepare some notations and terminologies.

- Given $\alpha \in (-1, 1)$, denote

$$\mathcal{M}_\alpha := \{m(x) \mid m \text{ satisfies (A) and (A1)}\}.$$

- We say $m \in \mathcal{M}_\alpha$ is *of bang–bang type* if there exists a measurable set $E \subset \Omega$ such that $m = \chi_E - \chi_{\Omega\setminus E}$, where χ_E is the characteristic function of E.
- Denote

$$\mathcal{B}_\alpha := \{m(x) \mid m \in \mathcal{M}_\alpha,\ \bar{m} = \alpha \text{ and } m \text{ is of bang–bang type}\}.$$

First, to study Questions 1.1, 1.2, we establish one result concerning the existence of positive solutions to the stationary problem of (1.1):

$$d\left[\bar{\theta} - \theta(x)\right] + \theta(x)[m(x) - \theta] = 0 \quad \text{in } \Omega. \tag{1.2}$$

Theorem 1.1. *Assume that $m(x)$ satisfies the assumption* (A). *Define*

$$\mu_0 = \mu_0(m) = \sup_{0 \ne \psi \in L^2(\Omega)} \frac{\int_\Omega \left(-d(\bar{\psi} - \psi)^2 + m(x)\psi^2\right) dx}{\int_\Omega \psi^2 dx}. \tag{1.3}$$

Then the problem (1.2)

$$d(\bar{\theta} - \theta) + \theta(m(x) - \theta) = 0 \quad \text{in } \Omega$$

admits a unique positive solution in $L^\infty(\Omega)$ if and only if $\mu_0 > 0$.

We remark that the same conclusion has been established in [1, Lemma 3.1] provided that $m(x) \in C(\bar{\Omega})$ using the upper/lower solution method. However, in Theorem 1.1, we only require $m(x) \in L^\infty(\Omega)$. Since $C(\bar{\Omega})$ is not dense in $L^\infty(\Omega)$, different method needs to be developed.

Moreover, due to Theorem 1.1, Question 1.1 is equivalent to the following question:

Question 1.1′. *Among all functions $m(x)$ that satisfy* (A) *and* (A1), *which $m(x)$ will yield the largest $\mu_0(m)$ defined in* (1.3)?

We will study Questions 1.1, 1.1′ separately using different approaches and obtain the following results respectively.

Theorem 1.2. *Assume that* (A) *and* (A1) *hold. For $m \in \mathcal{M}_\alpha$, define*

$$I(m) := \{d \mid d > 0 \text{ and for which (1.2) has a positive solution}\}.$$

Then if $m_\alpha \in \mathcal{B}_\alpha$, then for any $m \in \mathcal{M}_\alpha$, $I(m) \subset I(m_\alpha)$. Moreover,

$$I(m_\alpha) = \begin{cases} (0, +\infty), & 0 \le \alpha < 1, \\ (0, -1/\alpha), & -1 < \alpha < 0. \end{cases} \tag{1.4}$$