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Three dimensional non-isentropic subsonic Euler flows in rectangular nozzles

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ABSTRACT

In this paper, we study the existence and uniqueness of three dimensional non-isentropic subsonic Euler flows in rectangular nozzles. This work is an extension of Chen and Xie's work on isentropic subsonic Euler flows. If, besides small normal component of vorticity, Bernoulli's function and entropy function with small variations are given on the entrance, the existence and uniqueness of non-isentropic subsonic Euler flows are established.

1. Introduction and main results

Many phenomena in multidimensional gas flows are governed by the steady Euler system. A lot of progress has been made on the study of subsonic fluid since 1950s. One of the most important approximate model is the potential flow. The topic on subsonic potential flows past a bump was extensively studied by M. Shiffman [1], L. Bers [2,3], R. Finn, D. Gilbarg [4,5], and G.C. Dong [6]. Later, the existence of subsonic potential flows in infinitely long nozzles was established in [7–9]. Moreover, compensated compactness method was applied for investigation of subsonic–sonic flows in [10,7,8,11] and references therein. There were also many results on subsonic or subsonic–sonic potential flows in finitely long nozzles, see [12–15].

As a more physical model, the steady subsonic Euler flow is interesting for physicists and mathematicians. It gives rise to many challenging problems, too. The greatest feature for it is that the governing equations are not just an elliptic system but a hyperbolic–elliptic coupled system. The starting point for studying them can be found in [16] and later in [17]. They investigated steady incompressible Euler flows, which can be regarded as the zero Mach limits of subsonic flows. Since 2000s, two dimensional and three dimensional axially symmetric subsonic Euler flows have experienced a rapid development, see the Refs. [18–24] and so on. The key idea is to use a stream function formulation. However, such theory is only limited to a lower dimensional system and there is a long way for multidimensional subsonic Euler flows to be developed. Recently, C. Chen and C.J. Xie [25] gave the existence of three dimensional isentropic subsonic flows with small vorticity in rectangular nozzles via the transport equation for the vorticity and some div-curl system. S.K. Weng in [26] proved the existence of three dimensional subsonic Euler flows, which are small perturbations of the flows with straight horizontal streamlines.

Since the isentropic subsonic Euler flows are investigated in [25], the next natural problem is the existence of the non-isentropic subsonic Euler flows. Although there are some similar structures between isentropic flows and non-isentropic flows, the variation of entropy function makes the vorticity no longer transport invariant and the equations more complicated. In this paper, we study the non-isentropic subsonic Euler flows past a rectangular nozzle. The existence

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and uniqueness of non-isentropic flows are obtained when the normal component of the momentum is prescribed on the boundary and Bernoulli function, entropy function and the normal component of the vorticity are given at the entrance.

As is well known, the three dimensional steady non-isentropic compressible ideal flows are governed by the following Euler equations:

$$\mathrm{div}(\rho\mathbf{u}) = 0, \tag{1}$$

$$\mathrm{div}(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla p = 0, \tag{2}$$

$$\mathrm{div}(\rho\mathbf{u}S) = 0 \tag{3}$$

where ρ, $\mathbf{u} = (u_1, u_2, u_3)$, p, S are the density, velocity, pressure and entropy, respectively. It is assumed that the pressure p has the form

$$p = S\rho^\gamma \quad \text{for } \gamma > 1. \tag{4}$$

Eqs. (1)–(3) are hyperbolic–elliptic for subsonic flows ($|\mathbf{u}|^2 < c^2$) and hyperbolic for supersonic flow ($|\mathbf{u}|^2 > c^2$), where we call the quantity $c(\rho) = \sqrt{\frac{\partial p}{\partial \rho}(\rho, S)}$ local sound speed.

In this paper, the domain we consider is a rectangular domain $\Omega = [0, L] \times [0, 1]^2$ with the entrance $\Gamma_- = \{0\} \times [0, 1]^2$, the exit $\Gamma_+ = \{L\} \times [0, 1]^2$, and the solid wall $\Gamma = \partial\Omega \setminus (\Gamma_- \cup \Gamma_+)$.

In [25], it is proved that there exists a smooth subsonic potential flow past the rectangular nozzle, i.e.,

$$\begin{cases} \mathrm{div}(\rho(|\nabla\varphi|^2)\nabla\varphi) = 0 & \text{in } \Omega, \\ \rho(|\nabla\varphi|^2)\nabla\varphi\cdot\mathbf{n} = \bar{f} & \text{on } \partial\Omega \end{cases} \tag{5}$$

with $\rho(|\nabla\varphi|^2) = \left(\frac{\gamma-1}{\gamma S}(B - \frac{|\nabla\varphi|^2}{2})\right)^{\frac{1}{\gamma-1}}$ and $\mathbf{u} = \nabla\varphi$. Here B and S are given constants $\bar{B}$ and $\bar{S}$ in some suitable regions. The normal momentum $\bar{f} \in C^{2,\alpha}(\bar{\Omega})$ on the boundary satisfies the compatibility conditions

$$\int_{\partial\Omega} \bar{f}\,dS = 0, \tag{6}$$

$$\bar{f} < 0 \quad \text{on } \Gamma_-; \qquad \bar{f} = 0 \quad \text{on } \Gamma; \qquad \bar{f} > 0 \quad \text{on } \Gamma_+, \tag{7}$$

$$\frac{\partial\bar{f}}{\partial\nu} = 0 \quad \text{on } \partial\Gamma_- \cup \partial\Gamma_+, \tag{8}$$

where ν is the unit normal vector of $\partial\Gamma_-(\partial\Gamma_+)$ on $\Gamma_-(\Gamma_+)$. It is pointed out that the horizontal velocity is positive due to (5) and (7), that is

$$\min_{\bar{\Omega}} \bar{u}_1 = \min_{\bar{\Omega}} \partial_1\varphi > 0. \tag{9}$$

Moreover, (8) essentially guarantees the global regularity of the flows.

Therefore, the existence of isentropic Euler flows with small vorticity is obtained there, see [25]. Furthermore, if the entropy has small variation, the existence of non-isentropic Euler flows will be proved.

As considering potential flows, we prescribe the normal component of the momentum on the boundary, i.e.,

$$\rho\mathbf{u}\cdot\mathbf{n} = f, \tag{10}$$

where $\mathbf{n}$ is the unit outer normal vector and f satisfies the compatibility condition

$$\int_{\partial\Omega} f dS = 0, \qquad \frac{\partial f}{\partial\nu} = 0 \quad \text{on } \Gamma_- \cap \Gamma. \tag{11}$$

Furthermore, at the entrance Γ_- and the exit Γ_+, f satisfies

$$f < 0 \quad \text{on } \Gamma_-; \qquad f = 0 \quad \text{on } \Gamma; \qquad f > 0 \quad \text{on } \Gamma_+. \tag{12}$$

It is known that the three dimensional steady non-isentropic subsonic Euler system has two elliptic modes and three hyperbolic modes. Thus, additional conditions for hyperbolic factors should be imposed at the entrance. It follows from the Euler equations (1)–(3) that the Bernoulli function B defined by $B = \frac{|\mathbf{u}|^2}{2} + \frac{\gamma}{\gamma-1}S\rho^{\gamma-1}$ and the entropy function S are invariant along the streamlines, i.e.,

$$\mathbf{u}\cdot\nabla B = \mathbf{u}\cdot\nabla S = 0 \quad \text{in } \Omega. \tag{13}$$

Thus, at the entrance Γ_-, the following three boundary conditions are prescribed

$$(\nabla\times\mathbf{u})\cdot\mathbf{n} = \kappa(x_2, x_3), \qquad B = B_0(x_2, x_3) \qquad S = S_0(x_2, x_3) \quad \text{on } \Gamma_-. \tag{14}$$

Then our main result is as follows.