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The impact of water level fluctuations on a delayed prey–predator model

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ABSTRACT

Prey–predator interactions are influenced by many ecological factors. In this paper, a delayed periodic prey–predator model with instantaneous negative feedback is investigated to study the impact of water level on persistence of two fish populations living in an artificial lake. By using the continuation theorem of coincidence degree theory, and by constructing suitable Lyapunov functionals, a set of easily verifiable sufficient conditions is derived for the existence, uniqueness and global stability of positive periodic solutions to the model. Numerical simulation is carried out to illustrate the feasibility of our main results.

1. Introduction

The impact of water level fluctuations on the species communities has been widely studied in rivers, lakes and reservoirs [1–4]. Depending on the spatial and temporal extension water level fluctuations can influence the dynamics and structure of the fish community. Recently, in [5], the authors examine how seasonal variations in water level affect the outcome of prey–predator interactions in Pareloup lake in the south of France. The Pareloup lake is one of the five biggest artificial lakes in France, situated between Rodez and Millau (1260 ha, 168×10^6 m^3). Its maximum depth is 37 m, and mean depth is 12.5 m, allowing to store water during seasons of high electricity demand. The management of this lake is of considerable ecological importance. Significant variations of the water level of the lake can have a strong impact on the persistence of some species. In fact, the increase of water volume hinders the capture of the prey by the predator. The same reasoning is applied when there is a decrease in the volume of water, favoring the capture of the prey by the predator. The authors propose a new model to describe the interaction between roach species as prey and pike species as predator. The obtained results indicate that the water level has a qualitative effect on the dynamic behavior of the prey–predator system. Very recently, Moussaoui et al., in [6] investigated a more complex interaction among three species living in the Pareloup lake under seasonal succession; the authors showed that the system is permanent under some appropriate conditions and obtained sufficient conditions which ensure the existence of the positive 1-periodic solution (see [6]).

As pointed out by Freedman and Wu [7] and Kuang [8], it would be of interest to study the global existence of periodic solutions for systems with periodic delays. In order to reflect the dynamical behaviors of the models depending on the past

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information of the system, May [9] first proposed and discussed in 1973 briefly the delayed predator–prey system

$$\begin{cases} \dot{x} = x(t)\,(a_1 - a_{11}x(t-\tau) - a_{12}y(t)) \\ \dot{y} = y(t)\,(-a_2 + a_{21}x(t) - a_{22}y(t)) \end{cases} \tag{1}$$

where $x(t)$ and $y(t)$ can be interpreted as the population densities of the prey and the predator at time t, respectively, τ is the feedback time delay of the prey to the growth of the species itself, the parameters a_i and a_{ij} $(i, j = 1, 2)$ are all positive constants.

Since this remarkable work, delayed prey–predator models have been studied extensively by many researchers, and very rich dynamics have been observed (see, for example [10–29] and references cited therein).

Motivated by the above reasons, and considering that the delay may occur in the competition among preys; in this paper, we are concerned with the effects of the water level and time delays due to negative feedbacks on the global dynamics of the predator–prey systems. This is to look for conditions which guarantee the existence of a positive periodic solution of the system with the same period as the coefficients, such a solution describes an equilibrium situation consistent with the variability of environmental conditions and such that both populations survive. The trajectories in the phase plane of these solutions of the nonautonomous system take the place of the equilibria points of the autonomous system. The significance of the paper is that the conditions are related to the values of the accessibility function $r(t)$ which depends directly on the water level of the lake.

The organization of this paper is as follows. In the next section, a brief description of the model is presented, and the positivity of the solutions is proved. In Section 3, sufficient conditions are obtained for the existence of positive periodic solutions of the delayed prey–predator model by using Gains and Mawhin's continuation theorem of coincidence degree theory. In Section 4, by constructing a suitable Lyapunov functional, the uniqueness and global stability of positive periodic solutions of the delayed prey–predator model is our main concern. Two examples are given in Section 5 to illustrate the feasibility of our main results. A brief discussion is presented in Section 6.

2. Prey–predator model

Let $G(t)$ and $B(t)$ be respectively the biomass of the prey and predator at time t. When a predator attacks a prey, it has access to a certain quantity of food depending on the water level. When water level is low the predator is more in contact with the prey. Let $r(t)$ be the accessibility function for the prey. It is assumed that the function $r(t)$ is annual periodic and continuous, that is, r is 1-periodic. The minimum value r_1 is reached in spring and the maximum value r_2 is attained during autumn, denoted respectively by γ_G and γ_B the maximum consumption rate of the resource by the prey and predator. Let e_B be the conversion rate of the prey in biomass and m_G, m_B be respectively the consumption rate of biomass by metabolism of the prey and predator. The predator needs a quantity $\gamma_B B(t)$ for his food, but he has access to a quantity

$$r(t)\frac{G(t)B(t)}{B(t)+D}$$

here D measures the other causes of mortality outside the metabolism and predation. It gives the extent to which environment provides protection to the prey. If

$$\frac{r(t)G(t)}{B(t)+D} \geq \gamma_B$$

then the predator will be satisfied with the quantity

$$\gamma_B B(t)$$

for his food. Otherwise, i.e if

$$\frac{r(t)G(t)}{B(t)+D} \leq \gamma_B$$

the predator will content himself with

$$r(t)\frac{G(t)B(t)}{B(t)+D}.$$

Consequently, the quantity of food received by the predator is

$$\min\left(\gamma_B, r(t)\frac{G(t)}{B(t)+D}\right)B(t).$$

Accordingly, the delayed prey–predator model can be expressed as

$$\begin{cases} \dfrac{dG}{dt}(t) = G(t)\,(\gamma_G - m_G G(t-\tau(t))) - \min\left(r(t)\dfrac{G(t)}{B(t)+D}, \gamma_B\right)B(t) \\ \dfrac{dB}{dt}(t) = e_B \min\left(r(t)\dfrac{G(t)}{B(t)+D}, \gamma_B\right)B(t) - m_B B(t). \end{cases} \tag{2}$$