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A variational approach to the homogenization of laminate metamaterials

Hélia Serrano *

Departamento de Matemáticas, E.T.S.I. Industriales, Universidad de Castilla-La Mancha, 13071 Ciudad Real, Spain

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ABSTRACT

This paper proposes a variational approach to study the asymptotic behaviour of the magnetic induction response of composite materials with a laminate microstructure whose components have negative magnetic permeability and different positive electric permittivity. A procedure to compute explicitly the effective electric permittivity of composites made by alternate layers of two single-negative materials is presented.

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1. Introduction

Homogenization aims at describing mathematically the macroscopic behaviour of microstructures. Microstructures are structures in a fine length scale between the macroscopic scale and the atomic scale. Many microstructures may be seen in nature, but there are many others which come from industrial processes. Such an example is artificial composites, i.e. materials made by mixing different materials at small scales which happen to be homogeneous materials at macroscopic scale. See [1]. The properties of composites may be described by equations of classical physics at the microscopic length scale where the mixture of different components takes place. Nevertheless, their effective properties at macroscopic level may be obtained when the small length scales converge to 0. Homogenization comes in at this stage. See [2–7].

In the previous work [8], we have focused on general inhomogeneous anisotropic composites with periodic microstructure, whose magnetic properties at microscopic scale are described by the stationary Maxwell equations with periodic oscillatory coefficients. Precisely, we have obtained the effective magnetic properties at macroscopic scale through the homogenization of the vector potential formulation of Maxwell's equations in magnetism. These composites are called static since their properties do not change with time. See [9].

In this work, we want to go further and study the electromagnetic properties of a special type of composites: laminate metamaterials. Metamaterials are artificial composites with a periodic or non-periodic structure exhibiting extraordinary electromagnetic properties which cannot be found in natural composites. The term *metamaterial* was introduced by Rodger M. Walser, University of Texas at Austin, in 1999 to define initially *macroscopic composites having a synthetic, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation*. See [10,11]. Metamaterials may basically be divided into two categories: the electromagnetic (or photonic) crystals, and the effective media. The electromagnetic crystals are structures made of periodic micro- or nano-inclusions whose period is of the same order as the signal wavelength. The effective media are structures whose period is much smaller than the signal wavelength so that their electromagnetic properties can be described by using homogenization theory. See [12].

The electric permittivity (ε) and the magnetic permeability (μ) are two functions used to characterize, respectively, the electric and magnetic properties of materials interacting with electromagnetic fields. The majority of materials in nature

* Tel.: +34 635789787.
E-mail address: heliac.pereira@uclm.es.

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have positive permittivity and positive permeability so that they are called double-positive (DPS) media. On the other hand, materials with negative permittivity and negative permeability are named double-negative (DNG) media. However, so far they cannot be found in nature. In the middle range, materials with only one negative parameter are referred to as single-negative (SNG) media, and particularly they can be designated either as epsilon-negative (ENG) or mu-negative (MNG). For instance, cold plasma and silver have negative permittivities at microwave and optical frequencies, respectively, and ferromagnetic materials have negative permeability in the VHF and UHF regimes.

Let us consider an inhomogeneous composite with a periodic microstructure, with relative size $1/h$ ($h \in \mathbb{N}$), occupying a region Ω in $\mathbb{R}^3$, whose electric permittivity ε_h is a strictly positive space-dependent function while the magnetic permeability μ_h is a strictly negative one. The electromagnetic properties of such a medium at microscopic scale are modelled by the non-stationary Maxwell equations

$$\begin{cases} \partial_t D(x,t) - \operatorname{curl} H(x,t) = -J(x,t) \\ \partial_t B(x,t) + \operatorname{curl} E(x,t) = 0 \\ \operatorname{div} B(x,t) = 0 \\ \operatorname{div} D(x,t) = \rho(x,t) \end{cases} \tag{1.1}$$

in $\Omega \times (0,T)$, for some $T > 0$, where D and B stand for, respectively, the electric and magnetic induction, E and H stand for the electric and magnetic field, respectively, J is the given current density while ρ denotes the charge density. See [13]. This system of four equations with four unknowns may be reduced to a system with only two unknowns if we take into account the constitutive relations between the fields, i.e.

$$D(x,y) = \varepsilon_h(x)E(x,t)$$
$$H(x,t) = \mu_h^{-1}(x)B(x,t),$$

in $\Omega \times (0,T)$, where μ_h^{-1} stands for the inverse of μ_h. Namely, system (1.1) is equivalent to the following one

$$\begin{cases} \partial_t\left(\varepsilon_h(x)E(x,t)\right) - \operatorname{curl}\left(\mu_h^{-1}(x)B(x,t)\right) = -J(x,t) \\ \partial_t B(x,t) + \operatorname{curl} E(x,t) = 0 \\ \operatorname{div} B(x,t) = 0 \\ \operatorname{div}\left(\varepsilon_h(x)E(x,t)\right) = \rho(x,t), \end{cases} \tag{1.2}$$

whose unknowns are the electric field E and the magnetic induction B. Notice that, the fourth equation jointly with the first one lead us to the conservation of electric charge

$$\partial_t \rho(x,t) + \operatorname{div} J(x,t) = 0 \quad \text{in } \Omega \times (0,T).$$

Since we are considering a bounded region Ω with a perfectly conducting boundary $\partial\Omega$, the boundary conditions

$$E(x,t) \times \mathbf{n} = 0 \quad \text{and} \quad B(x,t)\cdot\mathbf{n} = 0 \quad \text{on } \partial\Omega \times (0,T), \tag{1.3}$$

where $\mathbf{n}$ stands for the unit outward normal vector to $\partial\Omega$, should be added to system (1.2).

Once the magnetic induction B is known, we may reconstruct the electric field E from the first equation of system (1.2) written as

$$\partial_t E(x,t) = \varepsilon_h^{-1}(x)\operatorname{curl}\left(\mu_h^{-1}(x)B(x,t)\right) - \varepsilon_h^{-1}(x)J(x,t), \tag{1.4}$$

whenever the electric permittivity ε_h has an inverse ε_h^{-1} and does not depend on time. Precisely,

$$E(x,t) = E(x,0) + \varepsilon_h^{-1}(x)\left[\operatorname{curl}\left(\mu_h^{-1}(x)\int_0^t B(x,\tau)\,d\tau\right) - \int_0^t J(x,\tau)\,d\tau\right]$$

for any $(x,t) \in \Omega \times (0,T)$. Thus, if we derive in time the second equation of system (1.2) and replace $\partial_t E$ by its expression in (1.4), system (1.2) coupled with the boundary conditions (1.3) reduces to

$$\begin{cases} \partial_t^2 B(x,t) + \operatorname{curl}\left(\varepsilon_h^{-1}(x)\operatorname{curl}\left(\mu_h^{-1}(x)B(x,t)\right)\right) = \operatorname{curl}\left(\varepsilon_h^{-1}(x)J(x,t)\right) & \text{in } \Omega \times (0,T) \\ \operatorname{div} B(x,t) = 0 & \text{in } \Omega \times (0,T) \\ B(x,t)\cdot\mathbf{n} = 0 & \text{on } \partial\Omega \times (0,T) \\ \varepsilon_h^{-1}\operatorname{curl}\left(\mu_h^{-1}B\right) \times \mathbf{n} = \varepsilon_h^{-1}J \times \mathbf{n} & \text{on } \partial\Omega \times (0,T) \end{cases} \tag{1.5}$$

where the magnetic induction B is the only unknown, and the given current density J is so that $\operatorname{div} J = -\partial_t \rho$. Regarding the initial and final data, we shall assume that

$$B(x,0) = B_0(x) \quad \text{and} \quad \partial_t B(x,0) = \partial_t B(x,T) = 0 \quad \text{in } \Omega, \tag{1.6}$$

given a divergence-free field B_0 in Ω, since we will treat this second-order initial–boundary value problem from a variational point of view.

Recall that, we are specially interested in describing the electromagnetic properties at macroscopic scale of a class of composites called laminates, i.e. materials with a microstructure formed by, at least, two materials placed in alternate layers

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