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Global existence and optimal decay rate of the compressible bipolar Navier–Stokes–Poisson equations with external force

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ABSTRACT

In this paper, we consider the three dimensional compressible bipolar Navier–Stokes–Poisson equations with the potential external force. Under the smallness assumption of the external force in some Sobolev space, the existence of the stationary solution is established by solving a nonlinear coupled elliptic system. Next, we show global well-posedness of the initial value problem for the three dimensional compressible bipolar Navier–Stokes–Poisson equations, provided the prescribed initial date is close to the stationary solution. Finally, based on the elaborate energy estimates for the nonlinear system and L^2-decay estimates for semigroup generated by the linearized equation, we give the optimal L^2-convergence rates of the solutions towards the stationary solution.

1. Introduction

In this paper, we consider the initial value problems of the three dimensional compressible bipolar Navier–Stokes–Poisson equations:

$$\begin{cases} \partial_t \rho_1 + \nabla\cdot m_1 = 0, \\ \partial_t m_1 + \nabla\cdot\left(\dfrac{m_1\otimes m_1}{\rho_1}\right) + \nabla p(\rho_1) = \rho_1\nabla\phi + \mu\Delta\left(\dfrac{m_1}{\rho_1}\right) + (\mu+\nu)\nabla\left(\nabla\cdot\left(\dfrac{m_1}{\rho_1}\right)\right) + \rho_1 F_1(x), \\ \partial_t \rho_2 + \nabla\cdot m_2 = 0, \\ \partial_t m_2 + \nabla\cdot\left(\dfrac{m_2\otimes m_2}{\rho_2}\right) + \nabla p(\rho_2) = -\rho_2\nabla\phi + \mu\Delta\left(\dfrac{m_2}{\rho_2}\right) + (\mu+\nu)\nabla\left(\nabla\cdot\left(\dfrac{m_2}{\rho_2}\right)\right) + \rho_2 F_2(x), \\ \Delta\phi = \rho_1 - \rho_2, \qquad \lim\limits_{|x|\to\infty}\phi(x,t) = 0, \\ (\rho_1, m_1, \rho_2, m_2)(x,0) = (\rho_{10}, m_{10}, \rho_{20}, m_{20})(x), \quad x\in\mathbb{R}^3. \end{cases} \tag{1.1}$$

The variable $t \geqslant 0$ is time, and $x \in \mathbb{R}^3$ is the spatial coordinate. The unknown functions are the particle densities $\rho_1 > 0$, $\rho_2 > 0$, the momenta m_1, m_2, and the electrostatic potential ϕ. The constants μ and ν are the viscosity coefficients satisfying $\mu > 0$ and $2\mu + 3\nu \geqslant 0$. $p = p(\rho_i)$ $(i = 1, 2)$ are the pressure functions satisfying $p'(\rho_i) > 0$ for $\rho_i > 0$. $F_1(x)$ and $F_2(x)$ are two given external force. The bipolar Navier–Stokes–Poisson system can be used to describe the motion of a compressible viscous isentropic Newtonian fluid in semiconductor devices or in plasmas. When only considering the dynamics of one particle in semiconductor devices and plasmas, we have the unipolar Navier–Stokes–Poisson equation. More details on the Navier–Stokes–Poisson equation can be found in [1–3].

Recently, some important progress was made for the unipolar Navier–Stokes–Poisson system. The local and/or global existence of renormalized weak solution for Cauchy problem of the multi-dimensional compressible Navier–Stokes–Poisson

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system were proved in [4–6]. Hao and Li [7], and Zheng [8] established the global strong solutions of the initial value problem for the multi-dimensional compressible Navier–Stokes–Poisson system in Besov space, respectively. The global existence and L^2-decay rate of the smooth solution of the initial value problem for the unipolar compressible Navier–Stokes–Poisson system in $\mathbb{R}^3$ were achieved by Li and his collaborators in [9,10]. Zhao and Li [11] studied the global existence and the optimal L^2-decay rate of solutions for the compressible unipolar Navier–Stokes–Poisson system with external force. The pointwise estimates of the smooth solutions for the three-dimensional isentropic compressible Navier–Stokes–Poisson equation were obtained in [12]. The quasineutral limit of the compressible unipolar Navier–Stokes–Poisson system was studied in [13–15]. Due to the interaction of two particles, there are very few studies on the bipolar Navier–Stokes–Poisson system. Duan and Yang [16] studied the unique existence and the asymptotic behavior of a smooth solution for the initial value problem for the one-dimensional bipolar Navier–Stokes–Poisson system, and Zhou and Li [17] showed the corresponding convergence rate. Li, et al. [18,19] established the global existence and L^2-decay rate of the smooth solution of the initial value problem for the bipolar compressible Navier–Stokes–Poisson system in $\mathbb{R}^3$. In this paper, we discuss the global existence and optimal L^2-decay rate of solutions for the compressible bipolar Navier–Stokes–Poisson system with external force (1.1), which can be regarded as a counterpart for the results in [18,19].

At the first step, we consider the potential force, namely, $F_1 = -\nabla\psi_1, F_2 = -\nabla\psi_2$. Introducing the velocity $u_1 = \frac{m_1}{\rho_1}, u_2 = \frac{m_2}{\rho_2}$, the problem (1.1) can be written as

$$\begin{cases} \partial_t\rho_1 + \nabla\cdot(\rho_1 u_1) = 0, \\ \partial_t u_1 + (u_1\cdot\nabla)u_1 + \dfrac{\nabla p(\rho_1)}{\rho_1} = \nabla\phi + \dfrac{\mu}{\rho_1}\Delta u_1 + \dfrac{\mu+\nu}{\rho_1}\nabla(\nabla\cdot u_1) - \nabla\psi_1, \\ \partial_t\rho_2 + \nabla\cdot(\rho_2 u_2) = 0, \\ \partial_t u_2 + (u_2\cdot\nabla)u_2 + \dfrac{\nabla p(\rho_2)}{\rho_2} = -\nabla\phi + \dfrac{\mu}{\rho_2}\Delta u_2 + \dfrac{\mu+\nu}{\rho_2}\nabla(\nabla\cdot u_2) - \nabla\psi_2, \\ \Delta\phi = \rho_1 - \rho_2, \qquad \lim\limits_{|x|\to\infty}\phi(x,t) = 0, \\ (\rho_1, u_1, \rho_2, u_2)(x,0) = (\rho_{10}, u_{10}, \rho_{20}, u_{20})(x), \quad x\in\mathbb{R}^3. \end{cases} \tag{1.2}$$

We assume that the initial date satisfies

$$(\rho_{10}, u_{10}, \rho_{20}, u_{20}) \to (\bar\rho, 0, \bar\rho, 0) \quad \text{as } |x|\to\infty,\ \bar\rho > 0.$$

Note that (1.1) and (1.2) are equivalent for smooth solutions with $\rho_1 > 0$, $\rho_2 > 0$. Our main purposes of the present paper are to show the global existence and asymptotic behavior of smooth solutions for (1.1)(or (1.2)) for the initial date around stationary solutions. We first study the following stationary problem

$$\begin{cases} \nabla\cdot(\tilde\rho_1\tilde u_1) = 0, \quad (\tilde u_1\cdot\nabla)\tilde u_1 + \dfrac{\nabla p(\tilde\rho_1)}{\tilde\rho_1} - \nabla\tilde\phi + \nabla\psi_1 - \dfrac{\mu\Delta\tilde u_1}{\tilde\rho_1} - \dfrac{(\mu+\nu)\nabla(\nabla\cdot\tilde u_1)}{\tilde\rho_1} = 0, \\ \nabla\cdot(\tilde\rho_2\tilde u_2) = 0, \quad (\tilde u_2\cdot\nabla)\tilde u_2 + \dfrac{\nabla p(\tilde\rho_2)}{\tilde\rho_2} + \nabla\tilde\phi + \nabla\psi_2 - \dfrac{\mu\Delta\tilde u_2}{\tilde\rho_2} - \dfrac{(\mu+\nu)\nabla(\nabla\cdot\tilde u_2)}{\tilde\rho_2} = 0, \\ \Delta\tilde\phi = \tilde\rho_1 - \tilde\rho_2, \qquad \lim\limits_{|x|\to\infty}\tilde\phi(x,t) = 0, \\ \tilde\rho_1\to\bar\rho, \qquad \tilde u_1\to 0, \qquad \tilde\rho_2\to\bar\rho, \qquad \tilde u_2\to 0, \quad \text{as } |x|\to\infty. \end{cases} \tag{1.3}$$

The following is our first main result on the existence and uniqueness of the stationary solutions.

Theorem 1.1. *There exists $\varepsilon_1 > 0$ such that if $\|(\Delta\psi_1, \Delta\psi_2)\|_3 \leqslant \varepsilon_1$, the problem (1.3) has a unique solution $(\tilde\rho_1, \tilde u_1, \tilde\rho_2, \tilde u_2, \tilde\phi)$ satisfying $\tilde\rho_1 - \bar\rho \in H^5(\mathbb{R}^3)$, $\tilde\rho_2 - \bar\rho \in H^5(\mathbb{R}^3)$, $\tilde u_1 = \tilde u_2 = 0$, $\nabla\tilde\phi \in H^4(\mathbb{R}^3)$, $\tilde\phi \in L^6(\mathbb{R}^3)$. If moreover, $\Delta\psi_1, \Delta\psi_2 \in L^1(\mathbb{R}^3)$, we have*

$$\|\tilde\rho_1 - \bar\rho\|_{L^1} + \|\tilde\rho_2 - \bar\rho\|_{L^1} \leqslant C\|(\Delta\psi_1, \Delta\psi_2)\|_{L^1}.$$

Remark 1.2. When there is no external force, it is easy to find some simple stationary solutions. However, we have to solve the nonlinear partial differential equations to find a stationary solution when the external forces appear. This is also much more complicated than the case for compressible Navier–Stokes equations with external forces where stationary solutions can be obtained easily through the inverse function of enthalpy, see [20].

Next, the global existence and asymptotic behavior of smooth solutions for (1.1) (or (1.2)) are stated as follows.

Theorem 1.3. *The exists $0 < \varepsilon_2 < \varepsilon_1$ such that if*

$$\|(\rho_{10} - \tilde\rho_1, u_{10}, \rho_{20} - \tilde\rho_2, u_{20})\|_3 + \|(\Delta\psi_1, \Delta\psi_2)\|_3 \leqslant \varepsilon_2. \tag{1.4}$$

the initial value problem (1.2) admits a unique solution $(\rho_1, u_1, \rho_2, u_2, \phi)(x,t)$ globally in time, which satisfies $\rho_1 - \tilde\rho_1, \rho_2 - \tilde\rho_2 \in C^0[0,\infty; H^3(\mathbb{R}^3)) \cap C^1[0,\infty; H^2(\mathbb{R}^3))$, $u_1, u_2 \in C^0[0,\infty; H^3(\mathbb{R}^3)) \cap C^1[0,\infty; H^1(\mathbb{R}^3))$, $\phi - \tilde\phi \in L^6(0,\infty; \mathbb{R}^3)$, $\nabla(\phi - \tilde\phi)$