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Existence and smoothness of continuous and discrete solutions of a two-dimensional shallow water problem over movable beds with nonlinear sediment transport relationship

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A B S T R A C T

A two-dimensional shallow water system over movable beds with nonlinear sediment transport relationship is considered in this paper. The existence of the solutions for the continuous system is proved here and their smoothness is investigated. A Galerkin method is employed to obtain a finite-dimensional problem. A Brouwer fixed point theorem is employed for this problem and it is shown that the model equations are satisfied by the limits of the resulting solution sequences.

We also consider the discretized problem using a local discontinuous Galerkin scheme. We perform an error analysis and show that the method is convergent and the error is bounded according to a specific norm defined herein.

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1. Introduction

Shallow water flows are observed in many natural cases such as rivers, lakes and coastal areas. The vertical scale of the flow in shallow currents is much less than the horizontal ones and physical properties of the flow have little change in the vertical direction compared with the horizontal one. In most situations, the bed materials are movable and because of erosion or sedimentation, the bathymetry changes in time. Solving shallow water over movable bed equations (SWMBE) is therefore important for engineering applications.

During the past two decades, the mathematical study of shallow water equations in terms of existence, uniqueness, and smoothness of the solutions in various conditions has been started and improved. Orenga [1] initially proved the existence of the solutions for the homogeneous system of shallow water equations (SWEs). That study was then extended to the non-homogeneous system of SWEs by Chatelon and Orenga [2].

Later, the smoothness and uniqueness of the solutions was studied by Chatelon and Orenga [3]. Munoz-Ruiz et al. then extended those existence, uniqueness, and smoothness results to the 1-D [4] and 2-D [4] bi-layer SWEs.

Emrullah and Teoman [5] in their paper applied a composite variational principle to a one-dimensional shallow water wave system in both plane and axisymmetric flows and for the dispersive shallow water wave systems. In particular, they concluded that shallow water wave systems in plane flows based on infinite Lie point symmetries admit infinite local conservation laws. An analysis of a low resolution model is studied in [6] with a rather limited physics and results have been obtained in specific cases. The author showed that the fourth order model in the square box allowed us to divide by two the forecast error of the 20 days forecast. The important role played by the boundary conditions on rigid boundaries is also outlined. In [7] the author showed that the Jungels existence result [8] still holds when the viscosity constant is bigger

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than the scaled Planck constant. Consequently, the author generalized results obtained in [9,8] and showed the existence for all physically interesting cases of the scaled Planck and viscosity constants. A two-component Degasperis–Procesi system which arises in shallow water theory is considered in [10]. The authors analyzed some aspects of the blow-up mechanism, traveling wave solutions and the persistence properties. They discussed the local well-posedness and blow-up criterion; a new blow-up criterion for this system with the initial odd condition has been established. The persistence properties of strong solutions have also been investigated.

Note that in [3,2,9,5–8,10,4,1] only SWEs have been considered.

Other authors investigated shallow water equations over movable beds. Indeed, a Godunov-type one-dimensional finite volume numerical model was developed in [11] for simulating dam break flow over mobile beds. The authors showed that, in the case of dam break flow over mobile beds, the upwind flux scheme, HLLC scheme and Roes scheme proved to be adequate for capturing the wave front of the simulated cases and the WAF scheme produced a slight spurious oscillation and over-estimated the scour depth. In the case of simulating unsteady flow in a natural river, they stated that the simulated surface and bed profiles showed reasonable agreement with data from the field survey, and are superior to the predictions of the HEC-RAS quasi-unsteady model. In [12] the authors presented a class of multilayer models for solving a one-dimensional morphodynamic problem. The model consisted of coupling the multilayer Saint-Venant equations for the hydrodynamics with the Exner equation for the morphodynamics. As numerical solvers they applied a robust finite volume method for the single-layer approach and a kinetic method for the multilayer approach. To verify the considered models they solved a wind driven flow in a closed basin. The obtained results exhibit completely different flow and sediment features. In [13] the authors discussed the asymptotic behavior of solutions to three-dimensional Navier–Stokes equations with nonlinear damping $|u|^{\beta-1}u$. They first studied the L2 decay of weak solutions with $\beta \geq 10/3$ by developing the classic Fourier splitting method. Second, for $7/2 \leq \beta < 5$, they proved the optimal upper bounds of the higher-order derivative of the strong solution by employing a new analysis technique. Finally, they investigated the asymptotic stability of the large solution to the system with $\beta \geq 7/2$ under large initial perturbation. Shuangcai and Christopher [14] presented a 2-D high-order model for fully coupled shallow water flow, nonequilibrium sediment transport, and bed evolution (PIHM-Hydro). A stable and second-order-accurate numerical algorithm was implemented on unstructured grids using an upwind finite volume method combined with a multidimensional gradient reconstruction and slope limiter technique. By using the approximate Riemann solver and the semi-implicit time integration technique based on CVODE, the numerical model allowed us to produce accurate and stable solutions over a wide range of spatial scales and hydrological events, such as discontinuous flow and the wetting–drying process. Toumbou and Mohammadian [15,16] studied coupled SWEs with a sediment transport equation (SWMBEs) where a linear relationship between the sediment discharge and flow velocity was considered. In [15] an existence theorem for the 1-D SWMBEs was established and the convergence of the associated numerical model using continuous and discontinuous Galerkin discretization was proved. In [16] a 2-D SWMBE was considered and theoretical existence results were established for linear sediment transport.

In this paper we study the two-dimensional SWMBEs for a nonlinear relationship between the sediment discharge and flow velocity which is the case for most natural circumstances and depends on the type of sediments and their physical characteristics such as density and mean diameter. We study and prove the existence and the smoothness of the solutions in suitable chosen spaces.

A continuous–discontinuous Galerkin scheme is also considered for numerical solution of the system. Using an error analysis approach, we show that the numerical solution converges to the analytical one. To the best of our knowledge, this paper is the first of its kind for two-dimensional flows over movable beds with nonlinear sediment transport.

This paper is structured as follows. The shallow water equations over movable beds are reviewed in Section 2. In Section 3 we present the weak formulation of the problem. The existence and the smoothness of the solutions in suitable chosen spaces are proved in Section 4. Finally, Section 5 deals with a continuous–discontinuous Galerkin scheme for numerical simulation of the equations and proves the convergence of the method. Some concluding remarks complete the study.

2. The 2-D shallow water equations over movable beds

The problem under study is the following 2-D SWMBE in a non-conservative form,

$$(\mathcal{P})\begin{cases}\partial_t\mathbf{u} - A\triangle\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} + g\nabla(h+z) = 0, & \textit{in } \mathbf{Q},\\ \partial_t h + \nabla\cdot(\mathbf{u}h) = 0, & \textit{in } \mathbf{Q},\\ \partial_t z + \sigma\nabla\cdot\mathbf{q}_s = 0, & \textit{in } \mathbf{Q},\\ \mathbf{u} = 0, & \textit{on } \partial\boldsymbol{\Omega}\times(0,T),\\ \mathbf{u}(t=0) = \mathbf{u}_0, & \textit{in } \boldsymbol{\Omega},\\ h(t=0) = h_0, & \textit{in } \boldsymbol{\Omega},\\ z(t=0) = z_0, & \textit{in } \boldsymbol{\Omega}.\end{cases} \tag{1}$$

The unknowns $h(\mathbf{x}, t)$, $\mathbf{u}(\mathbf{x}, t)$, and $z(\mathbf{x}, t)$ are defined on $\mathbf{Q} = \boldsymbol{\Omega}\times[0, T]$ and represent the flow depth, flow velocity, and the bed elevation at the section at the location of coordinate $\mathbf{x}$ at time t. Moreover, $\sigma = (1-\lambda_p)^{-1}$, where λ_p is the sediment porosity and A, g and, $q_s(u)$ are the constant coefficient of viscosity, the gravitational acceleration, and the sediment transport rate per unit width, respectively. The latter is a function of the flow velocity in the channel, which is written as

$$q_s(\mathbf{u}) = C_s\mathbf{u}\|\mathbf{u}\|^{m-1}, \tag{2}$$

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