Nonlinear Analysis: Real World Applications 14 (2013) 402–413

Contents lists available at SciVerse ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Influence of feedback controls on an autonomous Lotka–Volterra competitive system with infinite delays

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ARTICLE INFO

Article history:
Received 31 January 2012
Accepted 1 July 2012

Keywords:
Global stability
Feedback control
Extinction
Infinite delay
Lotka–Volterra

ABSTRACT

In this paper, we consider an autonomous Lotka–Volterra competitive system with infinite delays and feedback controls. The extinction and global stability of equilibriums are discussed using the Lyapunov functional method. If the Lotka–Volterra competitive system is globally stable, then we show that the feedback controls only change the position of the unique positive equilibrium and retain the stable property. If the Lotka–Volterra competitive system is extinct, by choosing the suitable values of feedback control variables, we can make extinct species become globally stable, or still keep the property of extinction. Some examples are presented to verify our main results.

1. Introduction and main results

The traditional Lotka–Volterra competition system has been studied extensively. Many excellent results concerned with the permanence, extinction and global attractivity of the Lotka–Volterra system are obtained, see [1–13] for example.

The following two-species autonomous competitive system

$$\begin{aligned} x_1'(t) &= x_1(t)(b_1 - a_{11}x_1(t) - a_{12}x_2(t)), \\ x_2'(t) &= x_2(t)(b_2 - a_{21}x_1(t) - a_{22}x_2(t)), \end{aligned} \tag{1.1}$$

where $b_i, a_{ij},\ i, j = 1, 2$ are positive constants, has been discussed in many books on mathematical ecology [14–16]. By the method of Lyapunov functions or see [4,13,17], if the coefficients of system (1.1) satisfy

$$\frac{a_{11}}{a_{21}} > \frac{b_1}{b_2} > \frac{a_{12}}{a_{22}}, \tag{H_1}$$

then system (1.1) has a unique positive equilibrium $(\bar{x}_1, \bar{x}_2) = \left(\frac{b_1a_{22}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{21}}, \frac{a_{11}b_2-b_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}}\right)$ which is globally attractive, i.e., all positive solutions of system (1.1) satisfy

$$\lim_{t\to+\infty} x_i(t) = \bar{x}_i, \quad i = 1, 2.$$

If the coefficients of system (1.1) satisfy

$$\frac{b_1}{b_2} > \frac{a_{11}}{a_{21}}, \qquad \frac{b_1}{b_2} > \frac{a_{12}}{a_{22}}, \tag{H_2}$$

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doi:10.1016/j.nonrwa.2012.07.004

then system (1.1) is extinct, that is, all positive solutions of system (1.1) satisfy

$$\lim_{t\to+\infty} x_1(t) = \frac{b_1}{a_{11}}, \qquad \lim_{t\to+\infty} x_2(t) = 0.$$

In [18], the authors argued that in a situation where the equilibrium is not the desirable one (or affordable) and a smaller value is required, we are required to alter the system structurally by introducing a feedback control variable [19] or [20]. This can be implemented by means of a biological control or some harvesting procedure so as to make the population stabilize at a lower value. A primary contribution of C.R. Darwin during the last century was the theory that feedback over long time periods is responsible for the evolution of species. In 1931 V. Volterra explained the balance between two populations of fish in a closed pond using the theory of feedback. Later, a series of mathematical models have been established to describe the dynamics of feedback control systems. On the other hand, ecosystems in the real world are continuously disturbed by unpredictable forces which can result in some changes of the biological parameters such as survival rates. In the language of control variables, we call the disturbance functions control variables.

Gopalsamy and Weng [17] introduced a feedback control variable into a two species competitive system and discussed the existence of the globally attractive positive equilibrium of the system with feedback controls. Hu et al. [21] considered the extinction of a nonautonomous Lotka–Volterra competitive system with pure delays and feedback controls. By simulation, the authors [21] found that suitable feedback control variables can transform extinct species into permanent species. For more details in this direction, please see [22–30].

Because of the restriction of their analysis technique, traditional works on the feedback control ecosystem only showed that feedback control variables play important roles in the persistence and stability properties of the system. Recently, Fan and Wang [31] considered the following logistic equation with feedback control

$$\begin{aligned} x'(t) &= x(t)\left(1 - \frac{x(t)}{K} - u(t)\right), \\ u'(t) &= -\eta u(t) + ax(t). \end{aligned}$$

By the technique of the upper and the lower solutions, the authors [31] showed that the feedback control has no influence on the global stability of the logistic equation with feedback control. In [32–36], they also showed that feedback control variables have no influence on the permanence or global stability of the corresponding systems.

The consideration of infinite delays goes back to the works of Volterra on biological growth models [37], where the cumulative effect of the past of a biological species during a time period τ (including the case $\tau = \infty$) was introduced by means of integral equations. Thus a more accurate model is $x'(t) = rx(t)(1-\frac{1}{K}\int_{-\infty}^{t} k(t-s)x(s)ds)$, where $k(t)$ is a weighting factor which says how much emphasis should be given to the size of the population at earlier times to determine the present effect on resource availability. For more details in this direction, please see [38,39].

Motivated by the above papers, for the system with infinite delays, how do feedback controls affect the global stability of the system? To find an answer to this question, we consider a two-species autonomous Lotka–Volterra competitive system with infinite delays and feedback controls. More precisely, in this paper, we study the following system

$$\begin{aligned} x_1'(t) &= x_1(t)\left(b_1 - a_{11}x_1(t) - a_{12}\int_0^{+\infty} K_1(s)x_2(t-s)ds - c_1u_1(t)\right), \\ x_2'(t) &= x_2(t)\left(b_2 - a_{21}\int_0^{+\infty} K_2(s)x_1(t-s)ds - a_{22}x_2(t) - c_2u_2(t)\right), \\ u_1'(t) &= -e_1u_1(t) + d_1x_1(t), \\ u_2'(t) &= -e_2u_2(t) + d_2x_2(t), \end{aligned} \tag{1.2}$$

where $b_i, a_{ij}, c_i, e_i, d_i,\ i, j = 1, 2$, are positive constants; $x_i(t)$ denotes the density of the population x_i; $u_i(t)$ denotes the feedback control variable.

The kernels $K_i : [0, +\infty) \to [0, +\infty),\ i = 1, 2$ are continuous functions such that

$$\int_0^{+\infty} K_i(s)ds = 1, \qquad \sigma_i = \int_0^{+\infty} sK_i(s)ds < +\infty, \quad i = 1, 2.$$

We assume that solutions of system (1.2) satisfy the initial conditions

$$x_i(\theta) = \varphi_i(\theta), \quad \theta \in (-\infty, 0], \qquad \varphi_i(0) > 0, \quad i = 1, 2, \tag{1.3}$$

where $\varphi_i,\ i = 1, 2$ are non-negative and bounded continuous functions on $(-\infty, 0]$.

Assume that

$$\frac{a_{11}e_1 + c_1d_1}{a_{21}e_1} > \frac{b_1}{b_2} > \frac{a_{11}}{a_{21}} > \frac{a_{12}}{a_{22}}. \tag{H_3}$$