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A novel LMI-based criterion for the stability of direct-form digital filters utilizing a single two's complement nonlinearity

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ABSTRACT

A criterion for the global asymptotic stability of direct-form digital filters using two's complement arithmetic is presented. The criterion is in the form of a linear matrix inequality (LMI) and, hence, computationally tractable. Splitting the two's complement nonlinearity sector $[-1, 1]$ into two smaller sectors, $[0, 1]$ and $[-1, 0]$, together with using a type of "generalized" sector condition by involving saturation nonlinearity, is the novel feature in the present proof. A special case of the criterion is highlighted. The effectiveness of the present approach is demonstrated by showing its ability to establish the two's complement overflow stability region, in the parameter space, for a second-order digital filter.

1. Introduction

The issues related to the stability of nonlinear continuous-time as well as discrete-time systems have generated a great deal of interest. For a sample of the literature on the issues, the reader is referred to [1–36] and the references therein.

When a linear time-invariant digital filter [37,38] is implemented on a digital computer or on special-purpose digital hardware, errors due to finite register length are unavoidable. The signals in digital filters are represented with finite precision arithmetic. Arithmetic operations (e.g., multiplications and additions) performed in the digital filter generally lead to an increase in the required word length. Therefore, precautionary measures have often to be taken for signal word length reduction, namely quantization and overflow correction. These word length reductions have the effect of inserting nonlinearities (quantization and overflow nonlinearities) in the digital filter. The quantization nonlinearity may be one of the following [39–41]: (1) a magnitude truncation quantizer, (2) a roundoff quantizer, (3) a value truncation quantizer. The overflow nonlinearity may, for example, be produced by employing one of the following overflow arithmetics [39–41]: (a) saturation arithmetic, (b) two's complement arithmetic, (c) triangular arithmetic, (d) zeroing arithmetic. As is well known, the nonlinearity produced by saturation arithmetic yields the least restrictive stability conditions. By contrast, the nonlinearity produced by two's complement arithmetic is a very strong nonlinearity and yields the most restrictive stability conditions. For further details on the generation of quantization and overflow nonlinearities in the implementation of digital filters using fixed-point arithmetic, the reader is referred to [39–41].

The above discussion reveals that a digital filter implemented with fixed-point arithmetic has a mathematical description in the form of a nonlinear difference equation [42].[1] The presence of nonlinearities may result in instability of the system. The limit cycle phenomenon, which is a characteristic of nonlinear systems, may possibly occur in the designed filter if the filter parameters are not chosen properly [19]. The zero-input limit cycles (or the so-called overflow oscillations)

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[1] As is well known, many physical systems and processes are modeled by difference equations [42]. Digital signal processing or the so-called discrete-time signal processing [37,38] is one area where difference equations are widely used.

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represent an unstable behavior and are undesirable in digital filters [19,25]. An excellent overview of the problems arising as a consequence of the occurrence of zero-input limit cycles can be found in [25]. In particular, as highlighted in [25], with a view to achieving an efficient design for a digital filter using fixed-point arithmetic, it is required to ensure that the filter is free of zero-input limit cycles. An important objective in the design of a digital filter is, therefore, to find the ranges of the values of the filter parameters and choose the values in such a way that the designed filter is free of zero-input limit cycles. If the zero solution of the system can be shown to be globally asymptotically stable, then this automatically implies the absence of limit cycles.

The issues related to the stability of digital filters have received considerable attention [18–36]. A number of criteria for the nonexistence of zero-input limit cycles in digital filters have been reported. Most of these criteria, in fact, imply a stronger result, namely global asymptotic stability. The stability for the various models of digital filter involving nonlinearities has been investigated in the literature. For example, [18–24] deal with direct-form digital filters involving a single nonlinearity. On the other hand, state-space digital filters involving multiple nonlinearities are studied in [25–36]. For a review of the different forms of digital filters, the reader is referred to [37–41].

The quantization and overflow nonlinearities may interact with each other. However, if the total number of quantization steps is large or, in other words, the internal word length is sufficiently long, then the effects of these nonlinearities can be regarded as decoupled or noninteracting and can be investigated separately. Under this decoupling approximation, quantization effects may be neglected when studying the effects of overflow [39]. For further details on the decoupling approximation as well as on the effects of quantization and overflow, the reader is referred to [39].

The stability of direct-form digital filters employing a single saturation nonlinearity (i.e., the nonlinearity generated by employing saturation overflow arithmetic [39–41]) has been studied in a number of references: [18–24]. This paper deals with the stability problem for such systems pertaining to two's complement overflow arithmetic. Here it is worth pointing out that the hardware implementation of a two's complement arithmetic adder is simpler and less expensive than that of a saturation arithmetic adder [20]. The paper presents a new criterion for the two's complement overflow stability of such systems. The criterion is in the form of a linear matrix inequality (LMI) and, hence, computationally tractable. The effectiveness of the criterion is demonstrated by showing its ability to establish the two's complement overflow stability region, in the parameter space, for a second-order digital filter.

Remark 1. This paper deals with the stability of direct-form digital filters with a single two's complement nonlinearity. The stability of state-space digital filters with multiple two's complement nonlinearities has been investigated in a number of references: [25,26,35,36].

2. The system description

The system under consideration is given by

$$\left.\begin{aligned} &G(z) = h_1 z^{-n} + h_2 z^{-(n-1)} + \cdots + h_n z^{-1},\\ &y(r) = \text{output of } G(z),\\ &f(y(r)) = \text{input of } G(z). \end{aligned}\right\} \tag{1}$$

The nonlinearity characterized by

$$\left.\begin{aligned} &f(y(r)) = y(r) && \text{if } |y(r)| \le 1\\ &-1 \le f(y(r)) \le 1 && \text{if } y(r) > 1\\ &-1 \le f(y(r)) \le 1 && \text{if } y(r) < -1 \end{aligned}\right\} \tag{2}$$

is under consideration. Eq. (2) includes, among others, two's complement overflow arithmetic [39–41]. We will neglect the effects of quantization [39]. The following condition is assumed to hold:

$$z^n - h_n z^{n-1} - h_{n-1} z^{n-2} - \cdots - h_2 z - h_1 \neq 0, \quad \text{for all } |z| \ge 1. \tag{3}$$

Condition (3) implies that the underlying linear system (i.e., when $f(y(r))$ is replaced by $y(r)$) is stable, i.e., the infinite-precision counterpart of the system is stable.

3. The main result

Let

$$\mathbf{A}_1 \in R^{(n-1)\times n} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{n-1} \end{bmatrix}, \qquad \mathbf{h} = [h_1, h_2, \ldots, h_n]^T, \tag{4}$$

where $\mathbf{I}_{n-1}$ denotes the $(n-1) \times (n-1)$ identity matrix, $\mathbf{0}$ the null vector of appropriate dimension, and the superscript T the transpose of a vector or matrix. The main result of this paper is given in the following theorem.