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Soliton solutions via auxiliary function method for a coherently-coupled model in the optical fiber communications

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ABSTRACT

A coherently-coupled nonlinear Schrödinger system in the optical fiber communications, with the mixed self-phase modulation (SPM), cross-phase modulation (XPM) and positive coherent coupling terms, is studied through the bilinear method with an auxiliary function. Solutions for that system are found to be of two types: singular and non-singular ones, and the latter appear as the soliton-typed. Vector bright one- and two-solitons are derived with the corresponding phase-shift parameter constraints. In virtue of computerized symbolic computation and asymptotic behavior analysis, elastic collision mechanisms of such vector solitons are investigated. With the aid of graphical simulation, vector solitons are displayed to be of the single- or double-hump profiles. The formation and collision mechanisms of the vector bright solitons for that system are generated based on the combined effects of SPM, XPM and coherent coupling. Only elastic collisions of the vector solitons occur for that system, which is a distinctive feature amid those of other coherently-coupled nonlinear Schrödinger systems.

1. Introduction

In the nonlinear optical fiber communications, vector solitons can be governed by the coupled nonlinear Schrödinger (NLS) systems from the polarized optical waves in isotropic mediums [1–14]. Actually, the term "vector soliton" is rather broad and covers different types of multi-component solitons, e.g., the two-component vector solitons (spatial or temporal) via two orthogonally polarized components of a single optical field or two fields of different frequencies but the same polarization [1–7]. The coupled NLS systems have been proposed to support the stable optical solitons with the consideration of coupling between copropagating fiber modes through nonlinearities and nonuniformities of different types and the collisions between two waves of different frequencies/polarizations [4–11]. Furthermore, the coupled NLS systems with coupling terms arising from the different frequencies/polarizations in the applicative context are also under development in nonlinear optics [1]. To describe the left- and right-polarized modes of the propagating electromagnetic waves in optical fibers, the following integrable Manakov system [5–10] has been proposed and studied, i.e.,

$$\mathrm{i}\, u_{1X} \pm \frac{1}{2} u_{1TT} + (|u_1|^2 + |u_2|^2) u_1 = 0, \tag{1a}$$

$$\mathrm{i}\, u_{2X} \pm \frac{1}{2} u_{2TT} + (|u_1|^2 + |u_2|^2) u_2 = 0, \tag{1b}$$

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where u_1 and u_2 are the complex amplitudes of the pulse envelopes, X and T represent the normalized spatial and temporal coordinates, and the "+" or "−" sign before the dispersive terms denotes the anomalous or normal dispersive regime, respectively. In the anomalous dispersive regime, Eqs. (1) possess a bright-soliton solution, and in the normal, a dark [7]. N-soliton solutions for Eqs. (1) have been reported and the inelastic collision of bright solitons has been analyzed as well [8]. That collision behavior has been exploited in the construction of logic gates [9] and also leads to the possibility of multi-state logic [10].

In Eqs. (1), the terms $|u_1|^2u_1$ and $|u_2|^2u_2$ represent the self-phase modulation (SPM) effect, while $|u_2|^2u_1$ and $|u_1|^2u_2$ the cross-phase modulation (XPM) effect. Note that the XPM terms $|u_2|^2u_1$ and $|u_1|^2u_2$ also act as a type of the incoherent coupling [14]. For the multi-component NLS equations, there actually exist two categories of the coupling effects, namely, the incoherent and coherent [1]. Coherent collision occurs when the nonlinear medium is weakly anisotropic or weakly birefringent [1,15]. Coherently-coupled vector solitons possess some properties different from those of the incoherently [1,16]. Vector solitons discussed for the incoherently-coupled NLS equations are said to be the incoherently-coupled in the sense that the coupling is phase insensitive [1]. Vector solitons associated with the coherent coupling among the optical fields are of different formation mechanisms from those with the incoherent coupling, where the coupling depends on the relative phases of interacting fields [1,4,17]. In that sense, Eqs. (1) constitute a two-component nonlinear system with the incoherent coupling (i.e., the XPM $|u_2|^2u_1$ and $|u_1|^2u_2$) and so does its integrable N-component generalization [10,18].

Regarding to the coherent coupling effect, in the framework of optical-fiber communications [1,4,19–24], the following two sets of two-coupled NLS systems, which describe the simultaneous propagation of two optical pulses or beams with different frequencies or polarizations in the weakly anisotropic or weakly birefringent media, appear as [20–23]

$$\mathrm{i}\, q_{1z} + q_{1tt} + 2\left(|q_1|^2 + 2|q_2|^2\right)q_1 - 2\, q_2^2 q_1^* = 0, \tag{2a}$$

$$\mathrm{i}\, q_{2z} + q_{2tt} + 2\left(2|q_1|^2 + |q_2|^2\right)q_2 - 2\, q_1^2 q_2^* = 0, \tag{2b}$$

and [4,20,24]

$$\mathrm{i}\, q_{1z} + q_{1tt} + 2\left(|q_1|^2 + 2|q_2|^2\right)q_1 + 2\, q_2^2 q_1^* = 0, \tag{3a}$$

$$\mathrm{i}\, q_{2z} + q_{2tt} + 2\left(2|q_1|^2 + |q_2|^2\right)q_2 + 2\, q_1^2 q_2^* = 0, \tag{3b}$$

where q_1 and q_2 are the slowly varying envelopes of two interacting optical modes, the variables z and t, respectively, correspond to the normalized distance and time, the terms q_{1tt} and q_{2tt} represent the group velocity dispersion (GVD), $|q_1|^2q_1$ and $|q_2|^2q_2$ the SPM, while $|q_2|^2q_1$ and $|q_1|^2q_2$ the XPM. The last terms $q_2^2q_1^*$ and $q_1^2q_2^*$ particular to Eqs. (2) and (3) [compared with Eqs. (1)] gives rise to the coherent coupling governing the energy exchange between two axes of the fiber. Based on the fact that the ratio between coefficients of the incoherent and coherent coupling terms is −2, the coherent coupling in Eqs. (2) has been denoted as the "*negative coherent coupling*", while in Eqs. (3), the "*positive coherent coupling*" [23]. In Ref. [23], with the introduction of an auxiliary function, a bilinear system of Eqs. (2) has been obtained, degenerate and non-degenerate vector solitons have been derived under the corresponding phase parameter conditions, and the collision mechanisms of vector solitons associated with the negative coherent coupling have been revealed as well. In Ref. [22], Darboux transformation (DT) has been applied to Eqs. (2) with the observation of one- and two-peak vector bright solitons, and propagation/collision for those solitons have also been discussed. Eqs. (3) has been investigated via the bilinear method with the results there as the degenerate and non-degenerate solitons with their collision analysis [4].

In the present paper, we will focus on the following coherently-coupled NLS system [20,22]:

$$\mathrm{i}\, q_{1z} + q_{1tt} + 2\left(|q_1|^2 - 2|q_2|^2\right)q_1 - 2\, q_2^2 q_1^* = 0, \tag{4a}$$

$$\mathrm{i}\, q_{2z} + q_{2tt} + 2\left(2|q_1|^2 - |q_2|^2\right)q_2 + 2\, q_1^2 q_2^* = 0, \tag{4b}$$

which has been derived by means of the Ablowitz–Kaup–Newell–Segur technique [22]. Attention should be paid to these aspects: ($\mathcal{A}$) Compared with Eqs. (1), coherent coupling ($q_2^2q_1^*$ and $q_1^2q_2^*$) appear in Eqs. (4); ($\mathcal{B}$) Nonlinear coupling terms in Eqs. (2)–(4) are all determined via the third-order susceptibility tensor; ($\mathcal{C}$) Coefficients of the SPM, XPM and coherent coupling terms in Eqs. (4) are "particular", that is, positive SPM in Eq. (4a) and negative SPM in Eq. (4b), negative XPM in Eq. (4a) and positive XPM in Eq. (4b), and positive coherent coupling in both Eqs. (4a) and (4b) (based on the fact that the ratio between the coefficients of incoherent and coherent coupling terms is 2). Conserved quantities and Lax pair of Eqs. (4) have been given [22], which indicate the complete integrability of Eqs. (4). For Eqs. (4), bright vector one- and two-soliton solutions including the one- and two-peak solitons are further constructed via the iterative algorithm of a DT [22].

Optical soliton propagation (without any change of envelopes) and collision can directly influence the communication capacity and quality [1,3,4,23–26]. Thus, in line with Refs. [4,23], we will use a form of the Hirota bilinear method to solve Eqs. (4). With symbolic computation [25,27], explicit vector bright one- and two-soliton solutions will be computed, which are of the single- or double-hump profile (also named as the one- or two-peak ones, consistent with those in Ref. [22]).

We call the attention that some results on Eqs. (4) are different from those on Eqs. (2) and (3), which are mainly twofold: (i) under the corresponding phase-shift parameter constraints, vector bright solitons will be classified into two types, the singular and non-singular ones, but not the degenerate and non-degenerate ones for Eqs. (2) and (3) [4,23], and (ii) the