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Stability analysis of flocking for multi-agent dynamic systems

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ABSTRACT

This paper is focused on the stability analysis of flocking for multi-agent dynamic systems under weakened assumptions on the connectivity of graphs. Sufficient conditions guaranteeing the emergence of collective behavior of a group of dynamic agents are established. We also consider the case when there is a leader among group of agents.

1. Introduction

Flocking is a form of collective behavior in group of interacting agents with a common group objective. In recent years, many scientists from rather diverse disciplines including animal behavior, physics, biophysics, social sciences, and computer science have been interested in the emergence of flocking, swarming, and schooling in a large number of agents with local interactions, such as birds, fish, penguins, ants, and crowds [1–11].

Since Reynolds introduced three heuristic rules (i.e., separation, cohesion and alignment rules) that led to the creation of the first computer animation of flocking [12], flocking problems have been extensively addressed in [13–18], partly due to the wide applications of flocking in many engineering areas including massive distributed sensing using mobile sensor networks; self-assembly of connected mobile networks; automated parallel delivery of payloads; and performing military missions [13]. Recently, consensus problems have received a surge of interest among control scientists [19–27]. In most consensus problems for networked dynamic systems, the objective is distributed computation of a function via agreement.

We particularly mention a paper by Olfati-Saber, who provided a theoretical and computational framework for design and analysis of scalable flocking algorithms in [13]. For the case when a virtual leader exists, it is assumed that each agent in the group has the same information of the virtual leader at any time [13]. Such an assumption was weakened to only a small fraction of agents in the group having information of the virtual leader [15]. When a virtual leader does not exist in a group of agents, it is assumed that all agents form a cohesive flock for $t \geq 0$ in [13].

In this paper, we will first show that the cohesion condition given in [13] is implied by a mild connectivity condition between agents. That is, we prove that flocking behavior can also emerge if a joint path exists from one agent to any other agent across $[t_0, t_0 + T]$ for any $t_0 \geq 0$ and some $T > 0$ in the moving frame. When a leader rather than a virtual leader exists, we propose a flocking algorithm which leads to flocking among agents if there exists a joint path from the leader to any other agent across $[t_0, t_0 + T]$ for any $t_0 \geq 0$ and some $T > 0$ in the moving frame.

An important method in establishing the main results of this paper is the LaSalle Invariance Principle (LIP). When all agents form a cohesive flock for all $t \geq 0$, the application of the LIP is straightforward. However, under the assumption that there is only a joint path from one agent to any other agent across $[t_0, t_0 + T]$ for any $t_0 \geq 0$ and some $T > 0$, it is somewhat difficult to apply the LIP since the boundedness of trajectories of all agents requires a careful examination.

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2. Problem statement

Consider a group of dynamic agents moving in the Euclidean space $\mathbb{R}^m$ (e.g. $m = 2, 3$). The equation of motion for each agent takes the following form

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i \end{cases} \tag{1}$$

where $q_i, p_i \in \mathbb{R}^m$, $i \in \mathcal{V} = \{1, 2, \ldots, n\}$, denote the position and velocity vectors of agent i, $u_i \in \mathbb{R}^m$ is the control input.

Throughout this paper, let

$$q = \mathrm{col}(q_1, q_2, \ldots, q_n), \qquad p = \mathrm{col}(p_1, p_2, \ldots, p_n).$$

Denote the neighboring graph at time t by $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ which is an undirected graph with the set of nodes $\mathcal{V}$ and the set of edges

$$\mathcal{E}_q(t) = \{(i, j) \in \mathcal{V} \times \mathcal{V} : \|q_j(t) - q_i(t)\| < r\},$$

where $r > 0$ is the interaction showing that communication between agents is limited, and $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^m$. The spatial neighbors of agent i are denoted by

$$\mathcal{N}_i(q) = \{j \in \mathcal{V} : \|q_j - q_i\| < r\}.$$

A path from agent i to agent $j (j \neq i)$ at time t is a sequence of edges $(i, i_1), (i_1, i_2), \ldots, (i_k, j) \in \mathcal{E}(t)$, where $i_j \in \mathcal{V}$ for $j = 1, 2, \ldots, k$. A joint path from agent i to agent $j (j \neq i)$ across $[a, b]$ with $b > a \geq 0$ is a sequence of edges $(i, i_1) \in \mathcal{E}(t_1),\ (i_1, i_2) \in \mathcal{E}(t_2), \ldots, (i_k, j) \in \mathcal{E}(t_{k+1})$, where $a \leq t_1 \leq t_2 \leq \cdots \leq t_{k+1} \leq b$ and $i_j \in \mathcal{V}$ for $j = 1, 2, \ldots, k$. Based on the basic knowledge of undirected graphs, we may let $k \leq n-2$. On the other hand, we see that $\mathcal{G}(t)$ is connected for all $t \in [a, b]$ if and only if there exists a path from one agent to any other agent for any $t \in [a, b]$. However, $\mathcal{G}(t)$ is not necessarily connected for all $t \in [a, b]$ if there exists a joint path from one agent to any other agent across $[a, b]$.

Throughout this paper, the bump function $\rho_h(z)$, $h \in (0, 1)$ takes the form

$$\rho_h(z) = \begin{cases} 1, & z \in [0, h) \\ \dfrac{1}{2}\left[1 + \cos\left(\dfrac{z-h}{1-h}\pi\right)\right], & z \in (h, 1] \\ 0, & \text{otherwise.} \end{cases}$$

We first recall a distributed algorithm given in [13] as the following:

$$u_i = \sum_{j=1, j\neq i}^{n} \phi_\alpha(\|q_j - q_i\|_\sigma)\mathbf{n}_{ij} + \sum_{j=1}^{n} a_{ij}(q)(p_j - p_i), \quad i \in \mathcal{N} \tag{2}$$

where the σ-norm $\|\cdot\|_\sigma$ is defined as

$$\|z\|_\sigma = \frac{\sqrt{1 + \epsilon\|z\|^2} - 1}{\epsilon}$$

with the fixed parameter $\epsilon \in (0, 1)$;

$$\mathbf{n}_{ij} = (q_j - q_i)/\sqrt{1 + \epsilon\|q_j - q_i\|^2}$$

is a vector along the line connecting q_i to q_j; $\phi_\alpha(z)$ is an action function defined by

$$\phi_\alpha(z) = \rho_h(z/r_\alpha)\phi(z - d_\alpha),$$
$$\phi(z) = \frac{1}{2}[(a+b)\sigma_1(z+c) + (a-b)],$$

where $d_\alpha = \|d\|_\sigma$, $r_\alpha = \|r\|_\sigma$, $\sigma_1(z) = z/\sqrt{1+z^2}$ and $\phi(z)$ is an uneven sigmoidal function with parameters that satisfy $0 < a \leq b$, $c = |a-b|/\sqrt{4ab}$ to guarantee $\phi(0) = 0$; the nonnegative smooth pairwise potential function is defined by

$$\psi_\alpha(z) = \int_{d_\alpha}^{z} \phi_\alpha(s)ds;$$

$a_{ij}(q)$ is the element of the spatial adjacency matrix $A(q)$ which is defined as

$$a_{ij}(q) = \begin{cases} \rho_h(\|q_j - q_i\|_\sigma / r_\alpha), & j \neq i \\ 0, & j = i. \end{cases}$$