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A class of dynamic frictional contact problems governed by a system of hemivariational inequalities in thermoviscoelasticity[☆]

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Dedicated to the memory of Professor Zdzislaw Naniewicz

ABSTRACT

In this paper we prove the existence and uniqueness of the weak solution for a dynamic thermoviscoelastic problem which describes frictional contact between a body and a foundation. We employ the nonlinear constitutive viscoelastic law with a long-term memory, which includes the thermal effects and considers the general nonmonotone and multivalued subdifferential boundary conditions for the contact, friction and heat flux. The model consists of the system of the hemivariational inequality of hyperbolic type for the displacement and the parabolic hemivariational inequality for the temperature. The existence of solutions is proved by using recent results from the theory of hemivariational inequalities and a fixed point argument.

1. Introduction

Problems involving thermoviscoelastic contact arise naturally in many situations, particularly those involving industrial processes when two or more deformable bodies may come in contact or may lose contact as a result of thermoviscoelastic expansion or contraction. For this reason there is a considerable literature devoted to this topic. The first existence and uniqueness results for contact problems with friction in elastodynamics were obtained by Duvaut and Lions [1]. Later, Martins and Oden [2] studied the normal compliance model of contact with friction and showed existence and uniqueness results for a viscoelastic material. These results were extended by Figueiredo and Trabucho [3] to thermoelastic and thermoviscoelastic models. In these papers the authors used the classical Galerkin method combined with a regularization technique and compactness arguments. Recently dynamic viscoelastic frictional contact problems with or without thermal effects have been investigated in a large number of papers; see e.g. Adly et al. [4], Amassad et al. [5], Andrews et al. [6,7], Chau et al. [8], Han and Sofonea [9], Jarusek [10], Kuttler and Shillor [11], Migórski [12], Migórski and Ochal [13], Migórski et al. [14,15], Rochdi and Shillor [16] and the references therein.

In this paper we consider the frictional contact problem between a nonlinear thermoviscoelastic body and an obstacle. We suppose that the process is dynamic and the material is viscoelastic with long memory and thermal effect. Our main interest lies in general nonmonotone and possibly multivalued subdifferential boundary conditions. More precisely, it is supposed that on the contact part of the boundary of the body under consideration, the subdifferential relations hold, the

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first one between the normal component of the velocity and the normal component of the stress, the second one between the tangential components of these quantities and the third one between temperature and the heat flux vector. These three subdifferential boundary conditions are the natural generalizations of the normal damped response condition, the associated friction law and the well known Fourier law of heat conduction, respectively. For examples, applications and detailed explanations concerning the boundary conditions we refer the reader to Panagiotopoulos [17,18], Naniewicz and Panagiotopoulos [19], and Migórski et al. [15].

The thermoviscoelastic phenomena can be divided into three classes: static, quasistatic, and full dynamic. The quasistatic problems can be viewed as being of mixed elliptic–parabolic type, while the dynamic case is of mixed hyperbolic–parabolic type. The latter is more complicated, and we have in the literature only a few results concerning existence and uniqueness. We investigate a fully dynamic contact problem which consists of the energy-elasticity equations of hyperbolic type together with a nonlinear parabolic equation for the temperature. Because of the multivalued and multidimensional boundary conditions, the problem is formulated as a system of two coupled evolution hemivariational inequalities. All subdifferentials are understood in this paper in the sense of Clarke and are considered for locally Lipschitz, and in general nonconvex and nonsmooth superpotentials. The multivalued boundary conditions considered in (9) cover several types of boundary conditions, e.g. the nonmonotone normal compliance condition, the simplified Coulomb friction law, the nonmonotone normal damped response condition, the viscous contact with Tresca's friction law, the viscous contact with power-law friction boundary conditions, the normal damped response with time-dependent Tresca's friction, the normal damped response with power-law friction, the nonmonotone variants of the friction law of Coulomb as well as the sawtooth laws generated by nonconvex superpotentials. For more details, we refer the reader to [19,17,18,15]. We also note that when the superpotentials involved in the problem are convex functions, the hemivariational inequalities reduce to variational inequalities.

The goal of the paper is to provide the result on existence and uniqueness of a global weak solution to the system. The existence of solutions is obtained by combining recent results on the hyperbolic hemivariational inequalities [20,21,15,22,23] and the results on the parabolic hemivariational inequalities [24,25], and by applying a fixed point argument. In spite of the importance of the subject in applications, to the best of the authors' knowledge, the existence of solutions to the system of hemivariational inequalities in dynamic thermoviscoelasticity has been studied in very few papers [26–28]. In these papers, there is a coupling only between the displacement (and velocity) and the temperature in the constitutive law which is assumed to be linear. In contrast to the aforementioned papers, in the present paper we deal with the fully nonlinear constitutive relation and assume that the coupling appears not only in the constitutive law but also in the heat flux boundary condition on the contact surface. Finally, we note that for linear thermoelastic materials a system of hemivariational inequalities was formulated by Panagiotopoulos in Chapter 7.3 of [18]. However, the regularity hypotheses on the multivalued terms were quite unnatural and the data were assumed to be very regular (cf. Proposition 7.3.2 in [18]).

The content of the paper is as follows. After the preliminary material of Section 2, in Section 3 we present the physical setting and the classical formulation of the problem. In Section 4 we deliver the variational formulation of the mechanical problem and state our main existence and uniqueness result. The proof of the main result is provided in Section 5.

2. Preliminaries

In this section we introduce notation and recall some definitions and results needed in the sequel; cf. [9,29,15,17].

We denote by $\mathbb{S}^d$ the linear space of second order symmetric tensors on $\mathbb{R}^d$, $d = 2, 3$, or equivalently, the space $\mathbb{R}^{d\times d}_s$ of symmetric matrices of order d. We recall that the canonical inner products and the corresponding norms on $\mathbb{R}^d$ and $\mathbb{S}^d$ are given by

$$u \cdot v = u_i v_i, \qquad \|v\|_{\mathbb{R}^d} = (v \cdot v)^{1/2} \quad \text{for all } u = (u_i),\ v = (v_i) \in \mathbb{R}^d,$$
$$\sigma : \tau = \sigma_{ij}\tau_{ij}, \qquad \|\tau\|_{\mathbb{S}^d} = (\tau : \tau)^{1/2} \quad \text{for all } \sigma = (\sigma_{ij}), \tau = (\tau_{ij}) \in \mathbb{S}^d,$$

respectively. Here and below, the indices i and j run from 1 to d, and the summation convention over repeated indices is adopted.

Let Ω be an open bounded subset of $\mathbb{R}^d$ with a Lipschitz continuous boundary Γ and let ν denote the outward unit normal vector to Γ. We introduce the spaces

$$H = L^2(\Omega;\mathbb{R}^d), \qquad \mathcal{H} = \left\{\tau = (\tau_{ij}) \mid \tau_{ij} = \tau_{ji} \in L^2(\Omega)\right\}, \qquad \mathcal{H}_1 = \{\tau \in \mathcal{H} \mid \mathrm{Div}\,\tau \in H\}.$$

It is well known that the spaces H, $\mathcal{H}$ and $\mathcal{H}_1$ are Hilbert spaces equipped with the inner products

$$\langle u, v\rangle_H = \int_\Omega u \cdot v\,dx, \qquad \langle \sigma, \tau\rangle_{\mathcal{H}} = \int_\Omega \sigma : \tau\,dx, \qquad \langle \sigma, \tau\rangle_{\mathcal{H}_1} = \langle \sigma, \tau\rangle_{\mathcal{H}} + \langle \mathrm{Div}\sigma, \mathrm{Div}\,\tau\rangle_H,$$

where $\varepsilon : H^1(\Omega;\mathbb{R}^d) \to \mathcal{H}$ and Div: $\mathcal{H}_1 \to H$ denote the deformation and the divergence operator, respectively, given by

$$\varepsilon(u) = \big(\varepsilon_{ij}(u)\big), \qquad \varepsilon_{ij}(u) = \frac{1}{2}\,(u_{i,j} + u_{j,i}), \qquad \mathrm{Div}\,\sigma = (\sigma_{ij,j}).$$

An index that follows a comma indicates a derivative with respect to the corresponding component of the spatial variable $x \in \Omega$. Given $v \in H^1(\Omega;\mathbb{R}^d)$ we denote by $\gamma_0 v$ its trace on Γ, where $\gamma_0 : H^1(\Omega;\mathbb{R}^d) \to H^{1/2}(\Gamma;\mathbb{R}^d) \subset L^2(\Gamma;\mathbb{R}^d)$ is the