



On the global boundary stabilization of the Camassa–Holm equation



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ABSTRACT

In the present paper, the boundary stabilization for the Camassa–Holm equation, which describes a generalized formulation for the shallow water wave equation, on an interval is investigated. This is a natural first step towards developing methods for control of flows. We derive nonlinear boundary control laws that achieve global asymptotic stability. We consider both the viscous and the inviscid Camassa–Holm equation, using both higher order boundary control and Dirichlet boundary control.

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1. Introduction

Recently, Holm and Staley introduced the b-family PDEs that describe the balance between convection and stretching for small viscosity in the dynamics of a 1D nonlinear wave in fluids [1]:

$$m_t + um_x + bu_x m = \varepsilon m_{xx}, \quad u = g * m. \quad (1.1)$$

Here $u = g * m$ denotes $u(x) = \int_{-\infty}^{\infty} g(x-y)m(y) dy$ and the term um_x denotes the convection term, $bu_x m$ the stretching term, and εm_{xx} the viscosity term. The convolution relates velocity u to momentum density m by integration against the kernel $g(x)$. In [1], the authors studied the effects of the balance parameter b and kernel $g(x)$ of the solitary wave structures and investigated their interactions analytically for $\varepsilon = 0$ and numerically for small viscosity $\varepsilon \neq 0$. When (1.1) is restricted to the peakon case $g(x) = e^{-|x|/\alpha}$ with length scale α , $m = u - \alpha^2 u_{xx}$, Eq. (1.1) may be expressed solely in terms of the velocity $u(x, t)$ as (see [1–6])

$$u_t + (b+1)uu_x - \varepsilon u_{xx} = \alpha^2(u_{txx} + uu_{xxx} + bu_x u_{xx} - \varepsilon u_{xxxx}), \quad (1.2)$$

When $\alpha \rightarrow 0$, Eq. (1.2) reduces to

$$u_t + (b+1)uu_x - \varepsilon u_{xx} = 0, \quad (1.3)$$

which covers the usual Burgers equation either by rescaling dimensions, or by setting $b = 0$. For $b = 2$, (1.2) is the 1D version of the 3D Navier–Stokes-alpha model for turbulence (see [7]) as $\varepsilon \neq 0$. In [8], Krstic studied the global stabilization of the Burgers equation by boundary control. Volkwein has studied the optimal control of the Burgers equation using the Lagrangian-SQP method [9]. Volkwein discussed the instantaneous control of the Burgers equation [10]. The distributed control problem for the Burgers equation was studied in [11]. Zhu, Tian, and Zhao studied the optimal control of the KdV–Burgers equation and the sufficient nonlinear KdV–Burgers equation [12,13]. Armaou and Christofides studied the feedback control of the Kuramoto–Sivashinsky equation [14]. Ghattas and Bark studied the optimal control of 2D and 3D incompressible Navier–Stokes flows [15]. In [16–18], Chen et al. indicated that there is a connection between the solutions of the NS- α and

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turbulence. Specifically, the explicit steady analytical solution of the NS- α equations was found to compare successfully with empirical and numerical experimental data for mean velocity and Reynold's stresses for turbulent flows in pipes and channels. These comparisons led them to identify the NS- α equations with the Reynold's averaged Navier–Stokes equations. These comparisons also led them to suggest that the NS- α equations could be used as a closure model for the mean effects of subgrid excitations.

For $b = 2$, $\alpha \neq 0$ and $\varepsilon = 0$, it becomes the Camassa–Holm equation [19]. It has a bi-Hamiltonian structure and is completely integrable [20–25]. Also there is a geometric interpretation of Eq. (1.2) in terms of geodesic flow on the diffeomorphism group of the circle [26,27]. In fact, the system (1.2) is related to the first negative flow of the AKNS hierarchy via a reciprocal transformation. Its solitary waves are peaked [28]. They are orbitally stable and interact like solitons [29,30]. In [31], Ding and Tian researched the solution of 1D Navier–Stokes-alpha model, or, the dissipative Camassa–Holm equation and got the existence of the global attractor of the dissipative Camassa–Holm equation. Tian et al. derived the existence of the optimal control of the viscous Camassa–Holm equation under the homogeneous boundary conditions [32]. Glass discussed the controllability and asymptotic stabilization of the Camassa–Holm equation with an additional force term, supported in a nonempty interval, used as the control input [33]. More recently, in [34], Perrollaz studied the initial–boundary value problem and asymptotic stabilization of the Camassa–Holm equation on an interval used the returning method. In [35], by using the fixed-point technique, the authors gave a local time existence and uniqueness result, based on which they discussed the initial–boundary value problem and the asymptotic stabilization problem of the two-component Camassa–Holm Equation.

For $b = 3$, Eq. (1.1) becomes the Degasperis–Procesi equation [36], which is modeling the unidirectional propagation of shallow water waves over a flat bottom. Here $u(t, x)$ stands for the fluid velocity at time t in the spatial x direction. After the D–P equation appeared, the Cauchy problem and the initial–boundary value problem for the Degasperis–Procesi equation have been studied extensively, such as in [37–45]. For example, the author established the local well-posedness to the D–P equation with initial data $u_0 \in H^s(R)$, $s > \frac{3}{2}$ on the circle [42] and on the line [43], and derived the precise blow-up scenario and a blow-up result. The global existence of strong solutions and global weak solutions to the Degasperis–Procesi equation was shown in [46,47]. Similar to the Camassa–Holm equation, the Degasperis–Procesi equation has not only global strong solutions [48,46] but also blow-up solutions [48,46,49,50]. Apart from these, it has global entropy weak solutions in $L^1(R) \cap BV(R)$ and $L^2(R) \cap L^4(R)$, cf. [51] and references within. Recently, it has been demonstrated [52] that the Degasperis–Procesi equation [53] also can be recast as a geodesic equation on $\text{Diff}(S^1)$, although in this case the connection does not derive from an invariant metric [54]. The advantage of the Degasperis–Procesi equation in comparison with the KdV equation lies in the fact that the Degasperis–Procesi equation has peaked solitons and models wave breaking (by wave breaking we understand that the wave remains bounded while its slope becomes unbounded in finite time).

To the author's best knowledge, there has been no research on the boundary stabilization of the viscous Camassa–Holm equation through the inhomogeneous boundary control inputs. In the present paper, we would like to derive nonlinear boundary control laws that achieve global asymptotic stability (in a very strong sense). We consider both the viscous and the inviscid Camassa–Holm equation, using both higher order boundary control and Dirichlet boundary control.

2. Problem statement

We first consider the viscous Camassa–Holm equation, i.e. $b = 2$, $\alpha = 1$, $\varepsilon > 0$ is a constant,

$$u_t - u_{txx} + 3uu_x - 2u_xu_{xx} - uu_{xxx} - \varepsilon(u - u_{xx})_{xx} = 0, \quad (2.1)$$

issued from the initial condition $u(x, 0) = u_0 \in L^2(0, 1)$. Our objective is to achieve set point regulation:

$$\lim_{t \rightarrow \infty} u(x, t) = U_d, \quad \forall x \in [0, 1] \quad (2.2)$$

where U_d is a constant, while keeping $u(x, t)$ bounded for all $(x, t) \in [0, 1] \times [0, \infty)$. Without loss of generality we assume that $U_d > 0$. By defining the regulation error as $w(x, t) = u(x, t) - U_d$, we get the system

$$w_t - w_{txx} + 3ww_x + 3U_dw_x - 2w_xw_{xx} - ww_{xxx} - U_dw_{xxx} - \varepsilon(w - w_{xx})_{xx} = 0. \quad (2.3)$$

Let $y = w - w_{xx}$, Eq. (2.3) can be written as follows:

$$y_t + 2yw_x + 3U_dy_x + wy_x - U_dy_{xxx} - \varepsilon y_{xx} = 0. \quad (2.4)$$

We will approach the boundary control problem for Eq. (2.4) (or equivalently (2.3)) by both higher order boundary control

$$w_{tx}(0, t) = \psi_0, \quad w_{tx}(1, t) = \psi_1, \quad w_{xx}(0, t) = v_0, \quad w_{xx}(1, t) = v_1, \quad (2.5)$$

and Dirichlet boundary control

$$w(0, t) = \varphi_0, \quad w(1, t) = \varphi_1, \quad (2.6)$$

with boundary conditions

$$w_x(0, t) = 0, \quad w_x(1, t) = 0, \quad (2.7)$$

where ψ_0 , ψ_1 , v_0 , v_1 and φ_0 , φ_1 are scalar control inputs. The integrators separating ψ_0 from $u_x|_{x=0}$ and ψ_1 from $u_x|_{x=1}$ can be regarded as a part of actuator dynamics which prevent direct actuation via boundary values of u_x .

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