



Sulfate attack in sewer pipes: Derivation of a concrete corrosion model via two-scale convergence

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ABSTRACT

We explore the homogenization limit and rigorously derive upscaled equations for a microscopic reaction–diffusion system modeling sulfate corrosion in sewer pipes made of concrete. The system, defined in a periodically-perforated domain, is semi-linear, partially dissipative and weakly coupled via a non-linear ordinary differential equation posed on the solid–water interface at the pore level. First, we show the well-posedness of the microscopic model. We then apply homogenization techniques based on two-scale convergence for a uniformly periodic domain and derive upscaled equations together with explicit formulas for the effective diffusion coefficients and reaction constants. We use a boundary unfolding method to pass to the homogenization limit in the non-linear ordinary differential equation. Finally, we give the strong formulation of the upscaled system.

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1. Introduction

This paper treats the periodic homogenization of a semi-linear reaction–diffusion system coupled with a nonlinear ordinary differential equation arising in the modeling of the sulfuric acid attack in sewer pipes made of concrete. The concrete corrosion situation we are dealing with here strongly influences the durability of cement-based materials especially in hot environments leading to spalling of concrete and macroscopic fractures of sewer pipes. It is financially important to have a good estimate on the moment in time when such pipe systems need to be replaced, for instance, at the level of a city like Los Angeles, USA. To get good such practical estimates, one needs on one side good and easy-to-use macroscopic corrosion models to be used for a numerical forecast of corrosion, while on the other side one needs to ensure the reliability of the averaged models by allowing them to incorporate a certain amount of microstructure information. The relevant question is: *How much of this oscillatory-type information is needed to get a sufficiently accurate description of the heterogeneous medium?* Due to the complexity of possible shapes of the microstructure, averaging cement-based materials is far more difficult than averaging metallic composites, for instance, with rigorously defined well-packed structure. In this paper, we imagine our concrete piece to be made of a periodically-distributed microstructure. Based on this assumption, we provide here a rigorous justification of the formal asymptotic expansion performed by us (in [1]) for this reaction–diffusion scenario. Note that in [1] upscaled models are derived for a more general situation involving a locally-periodic distribution of perforations.¹

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¹ The word “perforation” is seen here as a synonym for “pore” or “microstructure”.

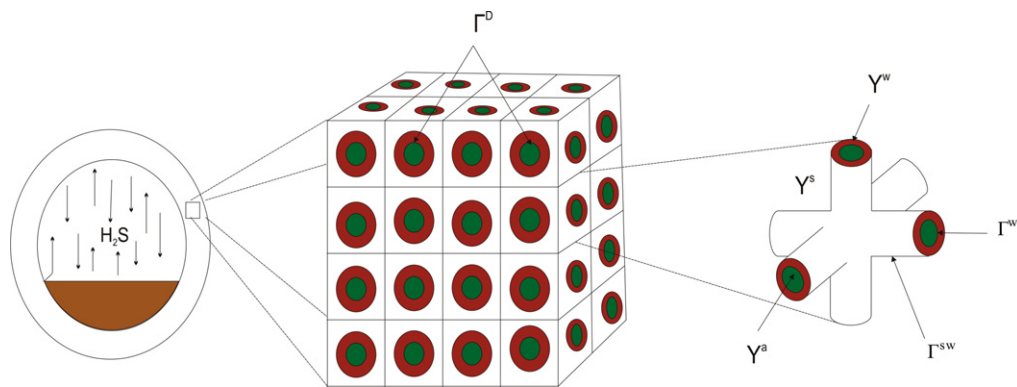


Fig. 1. Left: cross-section of a sewer pipe and pointing out domain of interest. Middle: periodic approximation of the periodic rectangular 3-dimensional domain. Right: reference pore configuration.

Locally periodic geometries refer to a special case of x -dependent microstructures, where, inherently, the outer normals to (microscopic) inner interfaces depend on both slow variable, say x , and fast variable, say y .

In the current framework, we combine two-scale convergence concepts with the periodic unfolding of ε -dependent interfaces to pass to the homogenization limit (i.e. to $\varepsilon \rightarrow 0$, where ε is a small parameter linked to the relative size of the perforation) for the uniformly periodic case. Here, the outer normals to the inner interfaces depend only on the spatial fast variable y . For more details on the mathematical modeling of sulfate corrosion of concrete, we refer the reader to [2,3] (a moving-boundary approach: numerics and formal matched asymptotics), [4] (a two-scale reaction–diffusion system modeling sulfate corrosion), as well as to [5], where a nonlinear Henry-law type transmission condition (modeling H_2S transfer across all air–water interfaces present in this sulfatation problem) is analyzed. Modeling and mathematics background on periodic homogenization can be found in e.g., [6–9], see also [10] (H -measures in the parabolic context), while a few relevant (remotely resembling) worked-out examples of this averaging methodology are explained, for instance, in [11–16]. It is worth noting that, since it deals with the homogenization of a linear Henry-law setting, the paper [13] is related to our approach. The major novelty here compared to [13] is that we now need to pass to the limit in a non-dissipative object, namely a nonlinear ordinary differential equation (ode). The ode is describing the sulfatation reaction at the inner water–solid interface—place where corrosion localizes. This aspect makes a rigorous averaging challenging. For instance, compactness-type methods do not work in the case when the nonlinear ode is posed on ε -dependent surfaces. We circumvent this issue by “boundary unfolding” the ode. Thus we fix, as independent of ε , the reaction interface similarly as in [17], and only then we pass to the limit using the monotonic structure of the production rate by reaction. Alternatively, one could use varifolds (cf. e.g. [18]), since this seems to be the natural framework for the rigorous passage to the limit when both the surface measure and the oscillating sequences depend on ε . However, we find the boundary unfolding technique easier to adapt to our scenario than the varifolds. To close the circle of discussion (at least for the periodic case), we apply in [19] periodic unfolding techniques to obtain corrector estimates for a reduced sulfate-corrosion problem.

Note that here we approach the corrosion problem deterministically. However, we have reasons to expect that the uniform periodicity assumption can be relaxed by assuming instead a Birkhoff-type ergodicity of the microstructure shapes and positions, and hence, the natural averaging context seems to be the one offered by random fields; see ch. 1, sect. 6 in [20], ch. 8 and 9 in [21], or [22]. But, methodologically, how big is the overlap between homogenizing deterministically locally-periodic distributions of microstructures compared to working in the random fields context? We will treat these and related aspects elsewhere. We expect the working techniques used in [23] to apply to our problem posed in a locally periodic setting.

The paper is organized as follows. We start off in Section 2 (and continue in Section 3) with the analysis of the microscopic model and derive ε -independent estimates. In Section 4, we extend the solution to the whole domain and give convergence results obtained by passage to the limit $\varepsilon \rightarrow 0$. Section 5 contains the main result of the paper: the set of the upscaled two-scale Eqs. (61)–(65).

2. The microscopic model

In this section, we describe the geometry of our array of periodic microstructures and briefly indicate the most aggressive chemical reaction mechanism typically active in sewer pipes. Finally, we list the set of microscopic equations.

2.1. Basic geometry

Fig. 1 (left) shows a cross-section of a sewer pipe hosting corrosion. We assume that the geometry of the porous medium in question consists of a system of pores periodically distributed inside the three-dimensional cube $\Omega := [a, b]^3$ with $a, b \in \mathbb{R}$ and $b > a$. The exterior boundary of Ω consists of two disjoint, sufficiently smooth parts: Γ^N – the Neumann

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