Nonlinear Analysis: Real World Applications 13 (2012) 686–693

Contents lists available at SciVerse ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Positive almost periodic solution for a class of Nicholson's blowflies model with a linear harvesting term

Fei Long *

Department of Mathematics and Computer Science, Hunan City University, Yiyang, Hunan 413000, PR China

ARTICLE INFO

Article history:
Received 12 February 2011
Accepted 9 August 2011

Keywords:
Positive almost periodic solution
Exponential convergence
Nicholson's blowflies model
Harvesting term

ABSTRACT

In this paper, we study the problem of positive almost periodic solutions for the generalized Nicholson's blowflies model with a linear harvesting term and multiple time-varying delays. By applying the fixed point theorem and the Lyapunov functional method, we establish some criteria to ensure that the solutions of this model converge locally exponentially to a positive almost periodic solution. Moreover, we give an example to illustrate our main results.

1. Introduction

It is well known that the population dynamics has traditionally been the dominant branch of mathematical biology, and the dynamics of populations are often described by delayed differential equations (DDEs). The application of DDEs in population dynamics dates back to the 1920s, when Volttera [1] investigated the predator–prey model. Now, the theory of the population dynamics has made a remarkable progress in the past ninety years with main results scattered in numerous research papers; see, for example, [2–4].

In the classic study of population dynamics, Gurney et al. [5] proposed the following nonlinear autonomous delay equation:

$$x'(t) = -\delta x(t) + Px(t-\tau)e^{-ax(t-\tau)}, \tag{1.1}$$

to describe the population of the Australian sheep-blowfly and to agree with the experimental data obtained in [6]. Here, $x(t)$ is the size of the population at time t, P is the maximum per capita daily egg production, $\frac{1}{a}$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. Since this equation explains Nicholson's data of blowfly more accurately, the model and its modifications have been now refereed to as Nicholson's Blowflies Model. Furthermore, the most important qualitative properties of the model such as existence of positive solutions, persistence, permanence, oscillation and stability have been intensively analyzed by numerous authors and some of these results can be found in [7–12].

As pointed out in [13,14], the periodically varying environment and almost periodically varying environment play important roles in many biological and ecological dynamical systems. Compared with periodic effects, almost periodic effects are more frequent. Moreover, many dynamic models with almost periodicity often appear in applied science and some practical problems concerning physics, mechanics, and the engineering technique fields (see [15–17]). Hence, there have been extensive results on the problem of the existence of almost periodic solutions for Nicholson's blowflies equation in the literature. We refer the reader to [18–21] and the references cited therein.

* Tel.: +86 07376353159; fax: +86 07376353159.
E-mail address: feilonghd@yahoo.com.cn.

1468-1218/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.nonrwa.2011.08.009

Recently, assuming that a harvesting function is a function of the delayed estimate of the true population, Berezansky et al. [22] presented an overview of the results on the classical Nicholson's model and proposed Nicholson's blowflies model with a linear harvesting term:

$$x'(t) = -\delta x(t) + px(t-\tau)e^{-ax(t-\tau)} - Hx(t-\sigma), \quad \delta, p, \tau, a, H, \sigma \in (0, \ +\infty). \tag{1.2}$$

Berezansky et al. [22] also pointed out an open problem: How about the dynamic behaviors of Nicholson's blowflies model with a linear harvesting term.

Now, a corresponding question arises: How about the existence and convergence of positive almost periodic of Nicholson's blowflies model (1.2). Motivated by the above, the main purpose of this paper is to give the conditions for existence and exponential convergence of the positive almost periodic solutions for Nicholson's blowflies model with a linear harvesting term. We consider the following generalized Nicholson's blowflies models with a linear harvesting term and multiple time-varying delays:

$$x'(t) = -\alpha(t)x(t) + \sum_{j=1}^{m} \beta_j(t)x(t-\tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))} - H(t)x(t-\sigma(t)), \tag{1.3}$$

where $t \in R, \alpha, H, \sigma, \beta_j, \gamma_j, \tau_j : R \longrightarrow [0, +\infty)$ are almost periodic functions, and $j = 1, 2, \ldots, m$. Obviously, (1.1)–(1.2) are the special cases of (1.3). On the other hand, as pointed in open problem 5 of [22], all the results on Nicholson's blowflies equations with linear impulsive perturbations are obtained for constant delays (see [9,19]). This implies that model (1.3) is more generally than those in the above literature.

For convenience, we introduce some notations. Throughout this paper, given a bounded continuous function g defined on R, let g^+ and g^- be defined as

$$g^- = \inf_{t\in R} g(t), \qquad g^+ = \sup_{t\in R} g(t).$$

It will be assumed that

$$\alpha^- > 0, \qquad \beta_j^- > 0, \qquad \gamma_j^- > 0, \quad j = 1, 2, \ldots, m, \tag{1.4}$$

and

$$r = \max\left\{\max_{1\le j\le m}\{\tau_j^+\},\ \sigma^+\right\} > 0. \tag{1.5}$$

We also suppose the following condition (S_1) hold.

(S_1) there exist two constants E_1 and E_2 such that

$$E_1 > E_2, \quad \sum_{j=1}^{m}\left(\frac{\beta_j}{\gamma_j}\right)^+ \frac{1}{\alpha^- e} < E_1, \qquad \sum_{j=1}^{m} \frac{\beta_j^-}{\alpha^+} E_1 e^{-\gamma_j^+ E_1} - \frac{H^+ E_1}{\alpha^+} > E_2 \ge \frac{1}{\min\limits_{1\le j\le m} \gamma_j^-}.$$

Let $C = C([-r, 0], R)$ be the continuous functions space equipped with the usual supremum norm $\|\cdot\|$, and let $C_+ = C([-r, 0], R_+)$ and $R_+ = [0, +\infty)$. If $x(t)$ is continuous and defined on $[-r+t_0, \sigma^*)$ with $t_0, \sigma^* \in R$, then, for all $t \in [t_0, \sigma^*)$, we define $x_t \in C$ where $x_t(\theta) = x(t+\theta)$ for all $\theta \in [-r, 0]$.

Due to the biological interpretation of model (1.3), only positive solutions are meaningful and therefore admissible. Thus we just consider the following initial conditions

$$x_{t_0} = \varphi, \quad \varphi \in C_+ \quad \text{and} \quad \varphi(0) > 0. \tag{1.6}$$

We write $x_t(t_0, \varphi)(x(t; t_0, \varphi))$ for a solution of the admissible initial value problem (1.3) and (1.6) with $x_{t_0}(t_0, \varphi) = \varphi \in C_+$ and $t_0 \in R$. Also, let $[t_0, \eta(\varphi))$ be the maximal right-interval of existence of $x_t(t_0, \varphi)$.

2. Preliminary results

In this section, some lemma and definition will be presented, which are of importance in proving our main results in Section 3.

Definition 2.1 (*See [13,14]*). Let $u(t) : R \longrightarrow R^n$ be continuous in t. $u(t)$ is said to be almost periodic on R if, for any $\varepsilon > 0$, the set $T(u, \varepsilon) = \{\delta : |u(t+\delta) - u(t)| < \varepsilon \text{ for all } t \in R\}$ is relatively dense, i.e., for any $\varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, such that for any interval with length $l(\varepsilon)$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $|u(t+\delta) - u(t)| < \varepsilon$, for all $t \in R$.

Set

$$B = \{\varphi | \varphi \text{ is an almost periodic function on } R\}.$$

For any $\varphi \in B$, if we define the induced modulus $\|\varphi\|_B = \sup_{t\in R} |\varphi(t)|$, then B is a Banach space.