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Ultimate boundedness and an attractor for stochastic Hopfield neural networks with time-varying delays

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ABSTRACT

This paper investigates ultimate boundedness and a weak attractor for stochastic Hopfield neural networks (HNN) with time-varying delays. By employing the Lyapunov method and the matrix technique, some novel results and criteria on ultimate boundedness and an attractor for stochastic HNN with time-varying delays are derived. Finally, a numerical example is given to illustrate the correctness and effectiveness of our theoretical results.

1. Introduction

Recently, it has been well recognized that stochastic disturbances are ubiquitous and inevitable in various systems, ranging from electronic implementations to biochemical systems, which are mainly caused by thermal noise, environmental fluctuations as well as different orders of ongoing events in the overall systems [1,2]. Therefore, considerable attention has been paid to investigate the dynamics of stochastic neural networks, and many results on stochastic neural networks with delays have been reported in the literature; see e.g. [3–18] and references therein. Among which, some sufficient criteria on the stability of uncertain stochastic neural networks were derived in [4–6]. Almost sure exponential stability of stochastic neural networks was discussed in [7–9]. In [10–14], mean square exponential stability and *p*th moment exponential stability of stochastic neural networks were investigated. Some sufficient criteria on the exponential stability of the periodic solution for impulsive stochastic neural networks were established in [15]. In [16], the stability of discrete-time stochastic neural networks was analyzed, while exponential stability of stochastic neural networks with Markovian jump parameters is investigated in [17,18]. However, these papers mainly concern the stability of stochastic neural networks.

In fact, except for the stability property, boundedness is also one of the foundational concepts of dynamical systems, which plays an important role in investigating the uniqueness of equilibrium, global asymptotic stability, global exponential stability, the existence of the periodic solution, its control and synchronization [19,20], and so on. Recently, ultimate boundedness of several classes of neural networks with time delays has been reported. Some sufficient criteria were derived in [21,22], but these results hold only under constant delays. Following, in [23], the globally robust ultimate boundedness of integro-differential neural networks with uncertainties and varying delays was studied. After that, some sufficient criteria on the ultimate boundedness of neural networks with both varying and unbounded delays were derived in [24], but the

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concerned systems are deterministic ones. In [25,26], a series of criteria on the boundedness, global exponential stability and the existence of the periodic solution for non-autonomous recurrent neural networks were established. To the best of our knowledge, there are few results on the ultimate boundedness and an attractor for stochastic neural networks. Therefore, the arising question about the ultimate boundedness and an attractor for the stochastic Hopfield neural networks with time varying delays is important yet meaningful.

The rest of the paper is organized as follows: some preliminaries are in Section 2, Section 3 presents our main results, a numerical example and conclusions will be in Sections 4 and 5, respectively.

2. Preliminaries

Consider the following stochastic HNN with time-varying delays

$$dx(t) = [-Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + J]dt + \sigma(x(t), x(t-\tau(t)))dw(t), \tag{2.1}$$

where $x = (x_1, \ldots, x_n)^T$ is the state vector associated with the neurons; $C = \mathrm{diag}\{c_1, \ldots, c_n\}$, $c_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when being disconnected from the network and the external stochastic perturbation; $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ represent the connection weight matrix and the delayed connection weight matrix, respectively; $J = (J_1, \ldots, J_n)^T$, J_i denotes the external bias on the ith unit; f_j denotes activation function, $f(x(t)) = (f_1(x_1(t)), \ldots, f_n(x_n(t)))^T$; $\sigma(\cdot,\cdot) \in R^{n\times m}$ is the diffusion coefficient matrix; $w(t)$ is m-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ generated by $\{w(s) : 0 \leq s \leq t\}$; $\tau(t)$ is the transmission delay and satisfies

$$0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq \mu. \tag{2.2}$$

The initial conditions are given in the form:

$$x(s) = \xi(s), \quad -\tau \leq s \leq 0,\ j = 1, \ldots, n,$$

where $\xi(s) = (\xi_1(s), \ldots, \xi_n(s))^T$ is $C([-\tau, 0]; R^n)$-valued function and $\mathcal{F}_0$-measurable R^n-valued random variable satisfying $\|\xi\|_\tau^2 = \sup_{-\tau\leq s\leq 0} E\|\xi(s)\|^2 < \infty$, $\|\cdot\|$ is the Euclidean norm and $C([-\tau, 0]; R^n)$ is the space of all continuous R^n-valued functions defined on $[-\tau, 0]$.

Throughout this paper, the following assumption will be considered.

(A1) There exist constants l_i^+ and l_i^- such that

$$l_i^- \leq \frac{f_i(x) - f_i(y)}{x - y} \leq l_i^+, \quad \forall x, y \in R.$$

It follows from [27] that under the assumption (A1), system (2.1) has a global solution on $t \geq 0$. We note that assumption (A1) is less conservative than that of in [3,6,28], since the constants l_i^+ and l_i^- are allowed to be positive, negative numbers or zeros.

The notation $A > 0$ (respectively, $A \geq 0$) means that matrix A is symmetric positive definite (respectively, positive semi-definite). A^T denotes the transpose of the matrix A. $\lambda_{\min}(A)$ represents the minimum eigenvalue of matrix A. Throughout the paper, all norms are assumed to be Euclid 2-norms.

3. Main results

In this section, we will give the conditions of the ultimate boundedness and then construct a compact set B_C as the weak attractor for the solutions by using the ultimate boundedness.

Theorem 3.1. *Suppose that there exist some matrices $P > 0$, $Q_i > 0$ $(i = 1, 2, 3, 4)$, $\sigma_1 > 0$, $\sigma_2 > 0$, $U_1 = \mathrm{diag}\{u_{11}, \ldots, u_{1n}\} \geq 0$, $U_2 = \mathrm{diag}\{u_{21}, \ldots, u_{2n}\} \geq 0$ and σ_3, such that*

(A2)

$$\Sigma = \begin{pmatrix} \Delta & \sigma_3 & PA + L_2U_1 & PB \\ * & \sigma_2 - (1-\mu)Q_1 - 2L_1U_2 & 0 & L_2U_2 \\ * & * & Q_3 + \tau Q_4 - 2U_1 & 0 \\ * & * & * & -(1-\mu)Q_3 - 2U_2 \end{pmatrix} < 0,$$

$$\begin{aligned} &\mathrm{trace}[\sigma^T(x(t), x(t-\tau(t)))P\sigma(x(t), x(t-\tau(t)))] \\ &\quad \leq x^T(t)\sigma_1 x(t) + x^T(t-\tau(t))\sigma_2 x(t-\tau(t)) + 2x^T(t)\sigma_3 x(t-\tau(t)), \end{aligned}$$

where $\Delta = Q_1 + \tau Q_2 + \sigma_1 - PC - CP - 2L_1U_1$, $L_1 = \mathrm{diag}\{l_1^- l_1^+, \ldots, l_n^- l_n^+\}$, $L_2 = \mathrm{diag}\{l_1^- + l_1^+, \ldots, l_n^- + l_n^+\}$, $*$ means the symmetric terms.