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Projective synchronization of different fractional-order chaotic systems with non-identical orders

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ABSTRACT

This paper investigates the projective synchronization (PS) of different fractional order chaotic systems while the derivative orders of the states in drive and response systems are unequal. Based on some essential properties on fractional calculus and the stability theorems of fractional-order systems, we propose a general method to achieve the PS in such cases. The fractional operators are introduced into the controller to transform the problem into synchronization problem between chaotic systems with identical orders, and the nonlinear feedback controller is proposed based on the concept of active control technique. The method is both theoretically rigorous and practically feasible. We present two examples that illustrate the effectiveness and applications of the method, which include the PS between two 3-D commensurate fractional-order chaotic systems and the PS between two 4-D fractional-order hyperchaotic systems with incommensurate and commensurate orders, respectively. Abundant numerical simulations are given which agree well with the analytical results. Our investigations show that PS can also be achieved between different chaotic systems with non-identical orders. We have further reviewed and compared some relevant methods on this topic reported in several recent papers. A discussion on the physical implementation of the proposed method is also presented in this paper.

1. Introduction

Since Pecora and Carroll's pioneer research work [1], chaos synchronization, as an important topic in nonlinear science, has been widely investigated in many fields. Due to the intrinsic character of chaotic system, chaos synchronization is believed to have a great variety of applications in physics [2], ecological systems [3], secure communications [4,5], etc. Up until now, people have observed and developed many forms of synchronization in chaotic systems, such as complete synchronization [1], phase synchronization [6], lag synchronization [7,8], generalized synchronization [9,10], projective synchronization [11–13] etc. (the reader is referred to [14] for more references). Amongst all kinds of chaos synchronization, projective synchronization (PS), characterized by a scaling factor that two systems synchronize proportionally, is the most noticeable one as the unpredictability of the scaling factor can enhance the security in digital communications.

In recent years, the study on the nonlinear dynamics and synchronization control of fractional-order chaotic systems has become a hot topic in nonlinear research area. Most of the above-mentioned synchronization forms in integer-order chaotic systems have also been observed in interacting fractional-order chaotic systems [15–19]. Till now, people have developed many effective methods to achieve the synchronization between identical or different fractional-order chaotic systems, such as Pecora–Carroll principle [20], linear control [21], linear state error feedback control [22], active sliding

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mode control [23], open-plus-closed-loop control [24] and active control [25]. One can see [26–28] for some most recent work on this topic. It should be mentioned that, like the studies on the synchronization of integer-order chaotic systems, the PS (and its modified forms) in fractional-order chaotic systems is still the dominant one among the synchronization research of various fractional-order systems.

In the aforementioned literature on chaos synchronization, the authors are all concerned with the synchronization research between chaotic systems of same orders; namely, the drive system and the response system are both of integer order or both of identical fractional orders. Recently, the synchronization problem between integer-order chaotic system and fractional-order chaotic system, or between fractional-order chaotic systems with different fractional derivative orders, began to attract the attention among researchers [29–35]. A detailed review about different methods on this topic can be found in [35]; in addition, a further comparison is elaborated in Section 5 of this paper. Since the synchronization study of chaotic systems with non-identical orders is a new subject in the research field of chaos synchronization, no other literature relevant to this topic have been present to the best of our knowledge. It should be noted that some investigations discussed in [36–42] etc. are not relevant to this topic though their titles indicate a seemingly similar meaning. The term *order*, as used in [36–42], means the number of the states in the chaotic system (namely, the *dimension* of the system) rather than the *derivative order* of the state variable. To name it more accurately, we would like to clarify that the research conducted in [36–42] just concerns on the conventional synchronization problem of integer-order chaotic systems with different dimensions.

Compared with the above-mentioned work [1,6–28,36–42] on the conventional chaos synchronization research, the synchronization scheme between chaotic systems with non-identical derivative orders apparently has some advantages. One of them is that the fractional orders of the states to be synchronized in the response fractional-order chaotic system are freed from the fixed orders of the corresponding states in drive system, which can provide more flexible mechanism in the selections of the drive and response systems. In addition, the fractional orders are variable parameters, which can be used as secret keys if this synchronization scheme is adopted in digital communications. In short, the research on the synchronization under such circumstance is a significant perspective, which can broaden the insight into the research on nonlinear fractional-order systems.

Motivated by the above discussions, this article investigates the PS between different fractional-order chaotic systems that can have different orders. Here we consider the case that the derivative orders of the state variables in response system are ***not greater*** than the orders of corresponding states in drive system. Based on some fundamental knowledge on fractional calculus, we propose a general approach to attain the PS between non-identical fractional-order systems. This method involves the fractional integral operators in the controllers, which is similar to the technique adopted in [33]. We also choose several typical examples to verify the effectiveness of the general method. Moreover, we have made a comparison on some available methods on this topic and a detailed discussion on the physical implementation of our method.

The rest of this paper is organized as follows. The basic properties of fractional calculus and essential theorems on the stability of fractional-order systems are introduced in Section 2. In Section 3, we discuss the problem formulation and then present a general method step by step to deal with this problem. In Section 4, we design controllers to achieve the PS of several typical fractional-order chaotic systems using the proposed method. Via credible numerical method, the corresponding simulation results are also given in this section. In Section 5, we review and compare some relevant methods reported from several recent papers; furthermore, we make a discussion on the feasibility of the physical implementation of our method. Finally, Section 6 concludes this paper.

2. Preliminaries

2.1. Some properties of fractional calculus

Throughout this paper we use the notation D^α as the simplified form of ${}_0^C D_t^\alpha (t > 0)$. In addition, as suggested in [43], we denote the fractional integral of order $\alpha > 0$ by $D^{-\alpha}$. In the following, we list some basic properties of fractional derivatives and integrals which are helpful in the proving analysis of the proposed method. For the detailed definitions and properties on fractional calculus, the reader is referred to [43,44].

(1) For $\alpha = n$, where n is an integer, the operation $D^\alpha f(t)$ gives the same result as classical calculus of integer order n. In particular, when $\alpha = 1$, the operation $D^1 f(t)$ coincides with the ordinary derivative $df(t)/dt$.

(2) For $\alpha = 0$, the operation $D^\alpha f(t)$ is the identity operation:

$$D^0 f(t) = f(t). \tag{1}$$

(3) Similar to integer-order calculus, fractional differentiation and fractional integration are linear operations:

$$D^\alpha [af(t) + bg(t)] = aD^\alpha f(t) + bD^\alpha g(t). \tag{2}$$

(4) For $\alpha \geq 0$, the following equation holds:

$$D^\alpha D^{-\alpha} f(t) = D^0 f(t) = f(t) \tag{3}$$

which means that the fractional differentiation operator is a left inverse to the fractional integration operator of the same order α.