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Reducing competitors in Cournot–Puu oligopoly

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A R T I C L E   I N F O

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The aim of this paper is to investigate whether an oligopoly given by isoelastic demand function and constant marginal costs converges to a duopoly, that is, all the firms except for two of them will not produce anything in future.

1. Introduction

It is well-known that Cournot oligopoly is a market model between monopoly and perfect competition (see e.g. [1,2] or [3]). In oligopoly, n different firms compete in a market producing the same good or perfect substitutes. When $n = 2$ we have a duopoly while for $n = 3$ the market is called a triopoly. Roughly speaking, the oligopoly is stated when we fix a demand function (which gives the price) and cost functions. Hence, firms react by maximizing their profits and obtain the best reply for each firm, which will be called reaction functions. In our case, these reaction functions for firm $i \in \{1, \ldots, n\}$ have the form $f_i(q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n)$, where q_i, $i = 1, \ldots, n$, are the outputs of each firm. Hence, the condition

$$q_i = f_i(q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n) \tag{1}$$

is satisfied for $i = 1, \ldots, n$. When we make the game dynamic, under naïve expectations, assume that the best reply at time $t + 1$ is given by the rule

$$q_i(t+1) = f_i(q_1(t), \ldots, q_{i-1}(t), q_{i+1}(t), \ldots, q_n(t)) \tag{2}$$

for $i = 1, \ldots, n$. The main aim for these systems is to understand all the possible trajectories, that is, all the possible solutions of the system of difference equations

$$\begin{cases} q_i(t+1) = f_i(q_1(t), \ldots, q_{i-1}(t), q_{i+1}(t), \ldots, q_n(t)), \\ q_i(0) = q_i^0, \end{cases} \tag{3}$$

for $q_i^0 \geq 0$ for $i = 1, \ldots, n$. In general, the above problem is quite difficult to solve and hence, we have to focus on partial approaches. One of them is to decide whether the number of competitors in the market can be reduced.

This problem was analyzed by Theocharis (see [4]), who considered an oligopoly generated by a linear demand function and constant marginal costs c_i, $i = 1, \ldots, n$. Theocharis's analysis of this model was just local. In fact, he analyzed the stability of the unique fixed or Cournot point of the system, finding that four oligopolists destabilize such a fixed point. To study how the number of firms is reduced we need to go further and analyze the global dynamics of the model. This was done in [5,6], where the conditions on the marginal costs of each firm for reducing the competitors to monopoly and duopoly

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were found. As one might expect, it was shown that when the number of competitors increases, the conditions for reducing firms from the market are more difficult to satisfy.

The aim of this paper is to analyze a similar question for the so-called Cournot–Puu oligopoly, in which the demand function is isoelastic and, again, the marginal costs are constant. The model is introduced in the next section, jointly with the basic mathematical background. The main results are proved in Sections 3 and 4.

2. The model; preliminaries

Consider a oligopoly of n firms with isoelastic demand function (in its inverse form)

$$p = \frac{1}{\sum_{i=1}^{n} q_i} = \frac{1}{Q}, \tag{4}$$

where p is the price, q_i, $i = 1, \dots, n$, is the output of each firm and Q is the total production. Assume also costs functions given by

$$C_i(q_i) = c_i q_i, \quad i = 1, \dots, n, \tag{5}$$

where c_i is constant for $i = 1, \dots, n$. Maximizing the profit functions

$$\Pi_i(q_1, \dots, q_n) = \frac{q_i}{Q} - c_i q_i, \quad i = 1, \dots, n, \tag{6}$$

we obtain the reaction functions

$$q_i = f_i(q_1, \dots, q_n) = \sqrt{\frac{Q_i}{c_i}} - Q_i, \quad i = 1, \dots, n, \tag{7}$$

where $Q_i = Q - q_i$ is the residual supply of each firm. If we make the game dynamic we have that each firm reacts following the maximal profit rule

$$q_i(t+1) = f_i(q_1(t), \dots, q_n(t)) = \max\left\{0, \sqrt{\frac{Q_i(t)}{c_i}} - Q_i(t)\right\}, \quad i = 1, \dots, n, \tag{8}$$

for $t \geq 0$. Note that outputs cannot be negative. Fixed points of the model are usually called Cournot points. The stability of such points is very important in oligopoly dynamics. When $n = 2$ we obtain the Cournot–Puu model introduced in [7] (see also [8]). In this case, the Cournot point $(\overline{q}_1, \overline{q}_2)$ (apart from the origin $(0, 0)$) is given by the equations

$$\begin{cases} \overline{q}_1^2 = \sqrt{\dfrac{\overline{q}_2^2}{c_1}} - \overline{q}_2^2, \\ \overline{q}_2^2 = \sqrt{\dfrac{\overline{q}_1^2}{c_2}} - \overline{q}_1^2, \end{cases} \tag{9}$$

which give us

$$\overline{q}_1^2 = \frac{c_2}{(c_1 + c_2)^2},$$
$$\overline{q}_2^2 = \frac{c_1}{(c_1 + c_2)^2}.$$

It is also proved in [7] that such a point is stable provided $r = c_2/c_1 \in (3 - 2\sqrt{2}, 3 + 2\sqrt{2})$. For $n = 3$, the local stability of the Cournot point was analyzed in [9,10], showing the existence of a Neimark–Sacker bifurcation. In this case, the Cournot point is given by the expression

$$\overline{q}_1^3 = 2\frac{-c_1 + c_2 + c_3}{(c_1 + c_2 + c_3)^2},$$
$$\overline{q}_2^3 = 2\frac{c_1 - c_2 + c_3}{(c_1 + c_2 + c_3)^2},$$
$$\overline{q}_3^3 = 2\frac{c_1 + c_2 - c_3}{(c_1 + c_2 + c_3)^2}.$$

Notice that, in the case of duopoly ($n = 2$), both $\overline{q}_1^2$ and $\overline{q}_2^2$ are positive. However, for triopoly ($n = 3$), $\overline{q}_1^3$ is negative provided $c_1 \geq c_2 + c_3$. So, it is natural to wonder whether, in this case, the triopoly evolves to a duopoly, that is, the first firm no longer produces and disappears from the market.