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Nonlinear stability of traveling wave fronts for delayed reaction diffusion systems[☆]

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ABSTRACT

This paper is concerned with nonlinear stability of traveling wave fronts for a delayed reaction diffusion system. We prove that the traveling wave front is exponentially stable to perturbation in some exponentially weighted L^∞ spaces, when the difference between initial data and traveling wave front decays exponentially as $x \to -\infty$, but the initial data can be suitable large in other locations. Moreover, the time decay rates are obtained by weighted energy estimates.

1. Introduction

In the past two decades, traveling wave fronts of delayed reaction–diffusion equations have been studied by many authors. Schaaf [1] studied two scalar reaction–diffusion equations with a discrete delay for both Huxley nonlinearity and Fisher nonlinearity and established the existence of traveling wave fronts and uniqueness of wave speeds by a phase plane analysis method. Wu–Zou [2] considered a more general reaction–diffusion system with finite delay and obtained the existence of traveling wave fronts by using the classical monotone iteration technique with sub-supersolution method. Using the method of Wu–Zou [2], Huang–Zou [3] established the existence of traveling wave fronts for a Lotka–Volterra cooperative system with delays. Ma [4] employed the Schauder's fixed point theorem to an operator used in [2] in a properly chosen subset in the Banach space $C(\mathbb{R}, \mathbb{R}^n)$ equipped with the so-called exponential decay norm, and showed the existence of traveling wave fronts for a class of delayed systems with quasimonotonicity reaction terms. Recently, Li et al. [5] developed a new cross iteration scheme, which is different from that defined in [4,2], and established the existence of traveling wave fronts for Lotka–Volterra competition system with delays. Traveling wave fronts for reaction–diffusion equations with nonlocal delay have also been studied by many authors, see [6–10] and the references therein.

The study of uniqueness and asymptotic stability of traveling wave fronts has become relatively more difficult. Sattinger [11] studied a reaction diffusion system without delay. By detail spectral analysis, he proved that the traveling wave fronts were stable to perturbations in some exponentially weighted L^∞ spaces. Kapitula [12] also studied a reaction diffusion system without delay. Using detail semigroup estimates, Kapitula [12] showed that the wave fronts is stable in polynomially weighted L^∞ spaces. By using Evans' method and detail semigroup estimates, Wu et al. [13,14] and Li–Wu [15] established the stability of traveling wave fronts for p-degree and double degenerate Fisher-type equations. Kan-on and Fang [16] obtained the asymptotic stability of monotone traveling waves for competition–diffusion equations without delay by using spectral analysis. The stability of the traveling wave for the Belousov–Zhabotinskii system was obtained by [17].

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For the stability and uniqueness of traveling wave fronts in reaction–diffusion equations with discrete delay, we should mention the work of Smith–Zhao [18]. They first established the existence and comparison theorem of a solution in a quasimontone reaction–diffusion bistable equation with a discrete delay and then obtained the stability of traveling wave fronts by using the elementary super-sub solution comparison and squeezing methods developed by Chen [19] (see also [20,8,9] for this technique). As far as we know, there is no result about the stability of traveling wave fronts for delayed reaction–diffusion systems.

Just recently, Mei et al. [21], Mei and So [22], Mei et al. [23]considered the so-called Nicholson's blowflies equation with diffusion

$$v_t - D_m v_{xx} + d_m v = \varepsilon b(v(x, t-r)), \tag{1.1}$$

where constants $D_m, d_m, \varepsilon > 0$, and

$$b(v) = pve^{-av^q} \quad \text{or} \quad b(v) = \frac{pv}{1+av^q}.$$

They first established a comparison principle and then proved that traveling wave fronts of the Eq. (1.1) are asymptotic stable in some exponentially weighted L^∞ spaces. Lv–Wang [24] considered some more general models and established the stability of traveling wave fronts using the method developed by Mei et al. [21].

Encouraged by papers [21–23], in this paper we use a similar method of [21,23] to study the stability of traveling wave fronts for the delayed Lotka–Volterra cooperative system. We first establish a new comparison principle and then prove that the traveling wave fronts of delayed Lotka–Volterra cooperative system are stable to perturbation in some exponentially weighted L^∞ spaces, when the difference between initial data and traveling wave front decays exponentially as $x \to -\infty$, but the initial data can be suitable large in other locations. Moreover, the time decayed rates are obtained. Our method is also suitable for the delay Lotka–Volterra competition system and delayed Belousov–Zhabotinskii system. For more information on the traveling wave front of Lotka–Volterra competition system and Belousov–Zhabotinskii system, see [2,25].

On the other hand, Mei et al. [21] and Lv–Wang [24] also considered the nonlocal nonlinearity. It is easy to see that our method is suitable for the following system

$$\begin{cases} u_t - d_1 u_{xx} = r_1 u \left(1 - a_1 u + b_1 \displaystyle\int_{\mathbb{R}} h_1(y) v(x-y, t-\tau) \mathrm{d}y\right), & (x,t) \in \mathbb{R} \times \mathbb{R}_+, \\ v_t - d_2 v_{xx} = r_2 v \left(1 + b_2 \displaystyle\int_{\mathbb{R}} h_2(y) u(x-y, t-\tau) \mathrm{d}y - a_2 v\right), & (x,t) \in \mathbb{R} \times \mathbb{R}_+ \end{cases}$$

with initial data (2.2), where $h_i(y) = \frac{1}{\sqrt{4\pi d_i}} e^{-\frac{y^2}{4r}}$, $i = 1, 2$. The stability of traveling wave fronts for delayed Lotka–Volterra competition system with nonlocal nonlinearity and delayed Belousov–Zhabotinskii system with nonlocal nonlinearity can be established similarly.

At last, it is well-known that $J * u - u$ can be considered as a generalized version of the Laplacian operator Δ, where $J \in C^1(\mathbb{R})$ is a nonnegative function with $\int_{\mathbb{R}} J(y)\mathrm{d}y = 1, J * u = \int_{\mathbb{R}} J(x-y)u(y)\mathrm{d}y$. The traveling wave fronts of nonlocal reaction–diffusion have been intensively studied by Levermore–Xin [26], Xin [27] and Pan et al. [28]. Pan et al. [28] considered the following nonlocal reaction–diffusion system with delay

$$u_t(x,t) = (J * u)(x,t) - u(x,t) + f(u(x,t), u(x, t-\tau)), \quad x \in \mathbb{R},\ t \geq 0, \tag{1.2}$$

where $u = (u_1, u_2, \dots, u_n)^T \in \mathbb{R}$. They obtained the existence of traveling wave fronts for system (1.2). It is easy to show that our method is also suitable for nonlocal reaction–diffusion system with delay. In our other paper [29], we have proved the stability of traveling wave fronts for nonlocal reaction–diffusion equation by using the same method.

Throughout this paper, $C > 0$ denotes a generic constant, while C_i $(i = 1, 2, \dots)$ represents a specific constant. Let I be an interval, typically $I = \mathbb{R}$. Denote by $L^2(I)$ the space of square integrable functions defined on I, and $H^k(I)$ $(k \geq 0)$ the Sobolev space of the L^2-functions $f(x)$ defined on the interval I whose derivatives $\frac{\mathrm{d}^i}{\mathrm{d}x^i} f$ $(i = 1, \dots, k)$ also belong to $L^2(I)$. Let $L^2_w(I)$ be the weighted L^2-space with a weight function $w(x) > 0$ and its norm is defined by

$$\|f\|_{L^2_w(I)} = \left(\int_I w(x)|f(x)|^2 \mathrm{d}x\right)^{\frac{1}{2}}.$$

Let $H^k_w(I)$ be the weighted Sobolev space with the norm given by

$$\|f\|_{H^k_w(I)} = \left(\sum_{i=0}^{k} \int_I w(x) \left|\frac{\mathrm{d}^i f(x)}{\mathrm{d}x^i}\right|^2 \mathrm{d}x\right)^{\frac{1}{2}}.$$

Let $T > 0$ be a number and B be a Banach space. We denote by $C^0([0, T]; B)$ the space of the B-valued continuous function on $[0, T]$, and by $L^2([0, T]; B)$ the space of the B-valued L^2-functions on $[0, T]$. The corresponding spaces of the B-valued L^2-functions on $[0, \infty)$ are defined similarly.