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The LMI method for stationary oscillation of interval neural networks with three neuron activations under impulsive effects[☆]

Fangfang Jiang [a,b], Jianhua Shen [a,*], Xiaodi Li [c]

[a] Department of Mathematics, Hangzhou Normal University, Hangzhou, Zhejiang 310036, China

[b] Department of Mathematics, Tongji University, Shanghai 200092, China

[c] School of Mathematical Sciences, Shandong Normal University, Jinan 250014, PR China

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ABSTRACT

In this paper, we investigate the stationary oscillation of interval neural networks with three neuron activation functions and time-varying delays under impulsive perturbations. Several theorems are given which present some sufficient conditions to guarantee the existence, uniqueness, and global exponential stability of periodic solutions (i.e., stationary oscillation) based on Lyapunov functionals approach and inequality analysis techniques. The obtained results can be easily checked by the Linear Matrix Inequality control toolbox in MATLAB. Finally, an example is given to illustrate the advantage of the obtained results.

1. Introduction

There are numerous examples of evolutionary systems which at certain instants of time are subjected to rapid changes. In the simulations of such processes, it is frequently convenient and valid to neglect the durations of the rapid changes and to assume that the changes can be represented by state jumps. Appropriate mathematical models for processes of the type described above are the so-called systems with impulsive effects. Significant progress has been made on the stability theory of systems of impulsive differential equations in recent decades. Similarly, delays (time-varying or constant) may occur also in the process of information storage and transmission in kinds of neural networks which can cause instability, oscillation and bad system performance [1,2]. Therefore, it is very necessary to study the case that the neural network models possess both delays and impulses. In recent years, various neural network models with time-varying delays and impulse effects have been extensively studied, particularly regarding dynamics analysis of equilibrium point [3–12]. It is well known that investigations on neural networks not only involve discussion of dynamics of equilibrium point, but also involve that of periodically oscillatory solutions. In many applications, the property of periodically oscillatory solutions are of great interest, see [3,13,14], such as, in [3], Li and Shen has studied stationary oscillation of interval neural networks with discrete and distributed time-varying delays under impulsive perturbations by employing some inequality analysis techniques and the LMI method. Moreover, the human brain has been in a periodic oscillatory or chaos state. Whence, it is of prime importance to study periodic oscillatory solutions of some neural network models. In addition, we know that the equilibrium point can be viewed as a special periodic solution with an arbitrary period; thus the analysis of periodic solutions is more general than that of the equilibrium point.

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* Corresponding author. Tel.: +86 571 28865286; fax: +86 571 28865286.
E-mail address: jhshen2ca@yahoo.com (J. Shen).

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On the other hand, the dynamics of a well-designed system may often be destroyed by its unavoidable uncertainty due to the existence of modeling error, external disturbance or parameter fluctuation during the implementation. In order to consider the effect of parameter fluctuation on neural networks, the dynamical analysis of interval neural networks was studied in [15–24]. For instance, Cui et al. [15] studied the global robust exponential stability for interval neural networks with time-varying delay by employing the Lyapunov stability theory and the linear matrix inequality (LMI) technique. In addition, switches between some different topologies are inevitable for many real-world dynamical networks due to abrupt changes of the network structure. Thus, the switched neural networks were proposed [25,26], such as in [26], the authors, by designing the full-order observers, investigate the global exponential stability problem for a class of switched recurrent neural networks with time-varying delay based on the average dwell time approach, the free-weighting matrix technique, and linear matrix inequalities. As we know, in hardware implementation of neural networks, the conditions to be imposed on the neural networks are determined by the characteristics of activation function as well as network parameters. When neural networks are designed to solving some practical problems, it is desirable for their activation functions to be general such as many electronic circuits, amplifiers which may generally have some different input–output functions [27,28]. To facilitate the design of neural networks, it is necessary and important that the neural networks with general activation functions are studied. To date, lots of sufficient conditions have been given for the existence, uniqueness, and global exponential stability of the periodic solution (i.e., stationary oscillation) of neural networks via different approaches; see [13,14,29–38], and the references therein. However, to the authors' best knowledge, there are few publications on stationary oscillation of interval neural networks with multiple different neuron activations via the LMI approach and inequality techniques.

In this paper, a class of interval neural networks with three neuron activations and time-varying delays under impulsive perturbations is discussed, in which external disturbance and parameter fluctuation are considered and the values of the parameters are not exactly known but bounded in given compact sets. In order to reflect a more realistic dynamic, we also consider the impulsive perturbations to the stationary oscillation of interval neural networks with three neuron activations. It is worth mentioning that the impulsive matrices in this paper are not necessarily diagonal, which makes its applications more extensive. Motivated by the idea of Li and Shen [3], by using inequality analysis and the Lyapunov functional method, we present some new conditions which ensure the stationary oscillation of interval neural networks with three neuron activations and impulses. The obtained results in this paper can be verified easily by the LMI Control Toolbox in MATLAB. In addition, it should be noted that the approach in this paper neither complicated Lyapunov–Krasovskii functionals which are used in [8,22], nor the continuation theorem of coincidence degree theory. The received LMI conditions in the present paper can be efficiently checked by the LMI Control Toolbox in MATLAB [39]. Furthermore, we observe that if the external input vector and time-varying delays become a constant vector and some constant delays, then the obtained conclusions can be viewed as byproducts of our main results (stationary oscillation). And, in this paper, we assumed neither differentiability nor monotonicity of the three different activation functions, so our results generalize those in [3] and improve and complement the results in [15,18–20,40].

2. Preliminaries and several lemmas

Notations. Let $\mathbf{R}$ denote the set of all real numbers, $\mathbf{R}_+ = [0, \infty)$, $\mathbf{Z}_+$ denote the set of positive integers and $\mathbf{R^n}$ the n-dimensional real space equipped with the Euclidean norm $|\bullet|$. $\mathbf{A} \geq 0$ or $\mathbf{A} \leq 0$ denotes that the matrix $\mathbf{A}$ is a symmetric and positive semi-definite or negative semi-definite matrix, $\lambda_{\max}(\mathbf{A})$ or $\lambda_{\min}(\mathbf{A})$ denotes the maximum eigenvalue or the minimum eigenvalue of matrix $\mathbf{A}$, and the notations $\mathbf{A}^T$ and $\mathbf{A}^{-1}$ mean the transpose of $\mathbf{A}$ and the inverse of a square matrix, respectively, $\Lambda \doteq \{1, 2, \dots, n\}$. $\mathbf{I}$ denotes the identity matrix with appropriate dimensions. For any interval $I \subseteq \mathbf{R}$, set $\Omega \subseteq \mathbf{R}^k (1 \leq k \leq n, k \in \mathbf{Z}_+)$, $PC(I, \Omega) \doteq \{\varphi : I \to \Omega$ is continuous everywhere except at some points t_k, $k \in \mathbf{Z}_+$, at which $\varphi(t_k^+) = \lim_{t \to t_k^+} \varphi(t)$, $\varphi(t_k^-) = \lim_{t \to t_k^-} \varphi(t)$ exist and satisfies $\varphi(t_k^+) = \varphi(t_k)\}$. Moreover, the notation $\star$ always denotes the symmetric block in one symmetric matrix.

In this paper, we consider impulsive interval neural networks with three neuron activations and time-varying delays of the form

$$\begin{cases} x'(t) = -\alpha(x(t)) + Af(x(t)) + Bg(x(t-\tau(t))) + W\displaystyle\int_{t-\mu(t)}^{t} h(x(s))ds + J(t), & t > 0,\ t \in [t_{k-1}, t_k), \\ \Delta x(t) = -D_k x(t^-), & t = t_k,\ k \in \mathbf{Z}_+, \\ x(s) = \varphi(s), & s \in [-\rho, 0], \end{cases} \tag{2.1}$$

where $\alpha(x(\cdot)) = (\alpha_1(x_1(\cdot)), \dots, \alpha_n(x_n(\cdot)))^T$ denotes the behaved function, $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ denotes the state vector associated with the neurons, and functions $f(x(\cdot)) = (f_1(x_1(\cdot)), \dots, f_n(x_n(\cdot)))^T$, $g(x(\cdot)) = (g_1(x_1(\cdot)), \dots, g_n(x_n(\cdot)))^T$, $h(x(\cdot)) = (h_1(x_1(\cdot)), \dots, h_n(x_n(\cdot)))^T$ denote the neuron activations, $n \geq 2$ is the number of units in a neural network; $J(t)$ is an external input vector, $\Delta x(t) = x(t) - x(t^-)$, $\{t_k\}$ is an impulsive sequence satisfying $0 = t_0 < t_k < t_{k+1} \uparrow \infty$ as $k \to \infty$; $\tau(t)$, $\mu(t)$ are the discrete time-varying delay and the distributed time-varying delay, respectively, and assumed to satisfy $0 \leq \tau(t) \leq \tau$, $0 \leq \mu(t) \leq \mu$, where τ and μ are two positive constants; define D_k, $k \in \mathbf{Z}_+$ to be some $n \times n$ real-valued impulsive matrices; $\rho \doteq \max\{\tau, \mu\}$; $\varphi(\cdot) \in PC([-\rho, 0], \mathbf{R^n})$. For $\varphi \in PC([-\rho, 0], \mathbf{R^n})$, the norm is defined by $\|\varphi\|_\rho = \sup_{-\rho \leq s \leq 0} |\varphi(s)|$. The matrices $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $W = (w_{ij})_{n\times n}$ are some unknown constant matrices.