Nonlinear Analysis: Real World Applications 10 (2009) 3322–3328

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Steady MHD flow of a third grade fluid in a rotating frame and porous space

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ARTICLE INFO

Article history:
Received 30 April 2008
Accepted 6 October 2008

Keywords:
Porous space
Third grade fluid
Slip condition

ABSTRACT

This paper looks at numerical solutions of steady state rotating and magnetohydrodynamic (MHD) flow of a third grade fluid past a rigid plate with slip. The space occupying the fluid is porous. The flow modeling is based upon the modified Darcy's law. The resulting non-linear problem is solved using MATLAB®. The influence of pertinent flow parameters on the velocity profiles is illustrated and discussed.

1. Introduction

Rotating flows of non-Newtonian fluids have many applications in meteorology, geophysics, turbomachinery and many other fields. Such flows in the presence of a magnetic field are significant because of their geophysical and astrophysical importance. MHD rotating fluids have relevance in the sunspot development, the solar cycle and rotating magnetic stars. In the literature much attention has been given to rotating flows of viscous fluids with no slip. Limited information is available for such flows involving non-Newtonian fluids [1–10]. Such investigations are further narrowed down if the slip condition is taken into account. More recently, Hayat and Abelman [11] examined slip effects on the flow in a rotating system and non-porous medium.

In general, the governing equations of non-Newtonian fluids are much more complicated, higher order and subtle in comparison to Navier–Stokes equations. The no-slip boundary conditions are insufficient to determine the unique solution, and additional conditions are required. Detailed critical reviews on the boundary conditions, the existence and uniqueness of the solution have been discussed at great length in the References [12–16]. We refer the interested reader also to the studies [17–21].

Flows of non-Newtonian fluids in a porous space have several engineering applications. These include flows in biomechanics, geothermal engineering, insulation systems, ceramic processing, enhanced oil recovery, etc. In the light of this, the purpose of the present communication is to extend the analysis discussed in [11] from a non-porous to a porous medium. The problem formulation is done by employing the modified Darcy's law. A non-linear expression is proposed for the modified Darcy's law. A numerical solution is obtained for a flow problem having a non-linear boundary value problem and non-linear boundary conditions. Due to slip effects, the boundary conditions are also non-linear. The slip condition means that the velocity of a fluid particle in the neighbourhood of the stationary plate is not the same as that of the plate. It is not necessary that the plate be porous. The slip condition is applied on the plate and not on the fluid, so that slip at the porous medium does not exist. In this study the slip condition is defined in terms of shear stresses. Salient features of the flow problem are illustrated graphically and discussed.

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doi:10.1016/j.nonrwa.2008.10.067

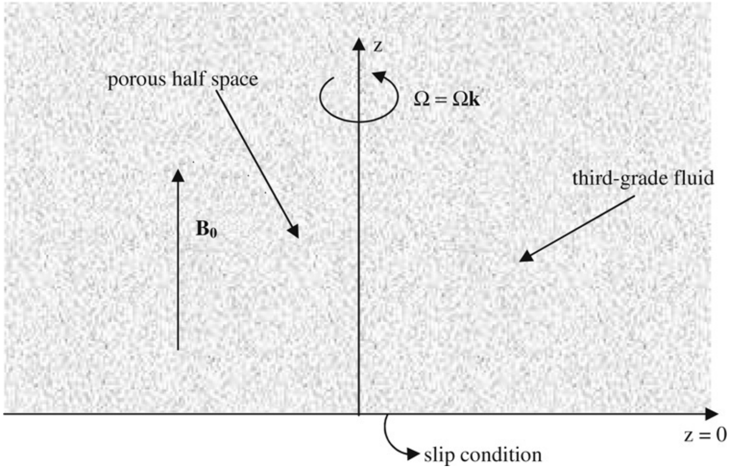

Fig. 1. Graphical representation of the problem.

2. Problem formulation

We consider a Cartesian coordinate system $\mathbf{r} = (x, y, z)$ with z-axis normal to the stationary rigid plate at $z = 0$. An incompressible third grade fluid fills the porous half space $z > 0$. The problem is depicted graphically in Fig. 1. The fluid conducts by applying a uniform magnetic field $\mathbf{B}_0$ parallel to the z-axis. The magnetic Reynolds number is taken small and hence the induced magnetic field is neglected. The applied and induced electric fields are assumed to be zero. Both the plate and fluid are in a state of solid body rotation with uniform angular velocity Ω about the z-axis. Under these assumptions the equations which govern the flow are:

$$\operatorname{div} \mathbf{V} = 0, \tag{1}$$

$$\rho \left[(\mathbf{V} \cdot \nabla) \mathbf{V} + 2\boldsymbol{\Omega} \times \mathbf{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) \right] = \operatorname{div} \mathbf{T} - \sigma B_0^2 \mathbf{V} + \mathbf{R}, \tag{2}$$

where ρ is the fluid density, σ is the electrical conductivity, $\mathbf{R}$ is the Darcy resistance of the fluid in a porous space and $\mathbf{V} = (u, v, w)$ is the velocity. Here u, v and w are the velocity components in the x-, y- and z-directions, respectively. In a third grade fluid, the expression for the Cauchy stress tensor satisfying the thermodynamic constraints [22]

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \quad \beta_1 = \beta_2 = 0, \qquad \beta_3 \geq 0,$$
$$|\alpha_1 + \alpha_2| \leq \sqrt{24 \mu \beta_3}$$

is given by [23]

$$\mathbf{T} = -p\mathbf{I} + \left(\mu + \beta_3 \left(\operatorname{tr} \mathbf{A}_1^2 \right) \right) \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2, \tag{3}$$

in which p is the scalar pressure, $\mathbf{I}$ is the identity tensor, μ is the dynamic viscosity and α_i $(i = 1, 2)$ and β_3 are the material constants. The first two Rivlin–Ericksen tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are [24]

$$\mathbf{A}_1 = (\nabla \mathbf{V}) + (\nabla \mathbf{V})^{\mathrm{T}},$$
$$\mathbf{A}_2 = \frac{\mathrm{d}\mathbf{A}_1}{\mathrm{d}t} + \mathbf{A}_1 (\nabla \mathbf{V}) + (\nabla \mathbf{V})^{\mathrm{T}} \mathbf{A}_1, \tag{4}$$

where $\mathbf{V}$ denotes the velocity field, ∇ is the gradient operator and $\frac{\mathrm{d}}{\mathrm{d}t}$ signifies the material time derivative.

It is well-known that Darcy's law provides an expression between the pressure drop induced by the frictional drag and the velocity. When there are boundaries of the porous medium, this law does not hold and the Brinkman law holds. For an Oldroyd-B fluid, the following expression of the modified Darcy's law has been proposed [25]:

$$\left(1 + \lambda \frac{\partial}{\partial t} \right) \nabla p = -\frac{\mu \, \phi}{k} \left(1 + \lambda_r \frac{\partial}{\partial t} \right) \mathbf{V}. \tag{5}$$

In the above equation λ and λ_r are the respective relaxation and retarding times, ϕ is the porosity and k is the permeability of the porous medium.

For Maxwell fluids, $\lambda_r = 0$ and Eq. (5) reduces to [26]:

$$\left(1 + \lambda \frac{\partial}{\partial t} \right) \nabla p = -\frac{\mu \, \phi}{k} \mathbf{V}. \tag{6}$$