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Adaptive anti-synchronization of chaotic complex nonlinear systems with unknown parameters

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ABSTRACT

This paper presents the adaptive anti-synchronization of a class of chaotic complex nonlinear systems described by a united mathematical expression with fully uncertain parameters. Based on Lyapunov stability theory, an adaptive control scheme and adaptive laws of parameters are developed to anti-synchronize two chaotic complex systems. The anti-synchronization of two identical complex Lorenz systems and two different complex Chen and Lü systems are taken as two examples to verify the feasibility and effectiveness of the presented scheme.

1. Introduction

Since Fowler et al. introduced the complex Lorenz equations [1], many complex chaotic systems have been proposed and studied in the last few decades. For example, Mahmoud et al. introduced the complex Chen and complex Lü systems and showed their chaotic attractors and the stability properties of their fixed points [2]. By adding a state feedback controller and using complex periodic forcing, a new hyperchaotic complex Lü system was constructed [3]. Another new chaotic complex nonlinear system was generated from the complex Lorenz system and its dynamical properties was also analyzed [4]. At the same time, an active control scheme is designed and applied to phase and antiphase synchronization of two identical hyperchaotic complex Lorenz systems [5]. In addition, two identical n-dimensional chaotic complex nonlinear systems are synchronized by an adaptive control scheme [6], just to enumerate a few examples. It is well known that the complex chaotic system also has much wider application. For example, the adoption of complex chaotic systems has been proposed for secure communication and the complex variables (doubling the number of variables) increase the contents and security of the transmitted information [2]. Another interesting phenomenon discovered was the anti-synchronization, which has been investigated both experimentally and theoretically in many physical systems [7–9].

Recently, the active control has been applied to anti-synchronize two identical chaotic real systems [10]. Moreover, it was examined in different types of chaotic real systems [11]. A nonlinear control scheme [12] and adaptive synchronization method [13] are also used to anti-synchronize two different chaotic real systems. However, methods of anti-synchronizing chaotic complex systems have not been extensively studied. Fortunately, some methods of anti-synchronizing chaotic real systems can be generalized to anti-synchronize chaotic complex systems. For example, active control techniques to anti-synchronize two identical chaotic complex systems with known parameters have been used in Ref. [5]. In fact, in practical engineering applications, it is hardly the case that every component of drive and response systems can be assumed to be identical and the system's parameters are exactly known in priori. Therefore, how to effectively anti-synchronize two different chaotic complex systems with unknown parameters is more essential and useful in real-life applications. The adaptive control scheme is an effective method to achieve the anti-synchronization of chaotic complex systems with fully unknown parameters.

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In this paper, we will focus on the anti-synchronization of a class of chaotic complex systems described by a unified mathematical expression. The adaptive control scheme will be employed. Based on Lyapunov stability theory, an adaptive controller and the adaptive laws of parameters will be developed to achieve the anti-synchronization. The adaptive anti-synchronizations of two identical and different chaotic complex systems are taken as examples to demonstrate the feasibility of the proposed control technique.

2. System mathematical models and problem descriptions

We consider a drive chaotic complex nonlinear system in the form of

$$\dot{\mathbf{z}} = \mathbf{F}(\mathbf{z})\mathbf{A} + \mathbf{f}(\mathbf{z}), \tag{1}$$

where $\mathbf{z} = [z_1, z_2, \ldots, z_n]^T$ is a state complex vector (T denotes transpose), and $\mathbf{z} = \mathbf{z}^r + j\mathbf{z}^i$. Define $z_1 = u_1 + ju_2, z_2 = u_3 + ju_4, \ldots, z_n = u_{2n-1} + ju_{2n}$, then $\mathbf{z}^r = (u_1, u_3, \ldots, u_{2n-1}), \mathbf{z}^i = (u_2, u_4, \ldots, u_{2n})^T, j = \sqrt{-1}$. $\mathbf{f} = (f_1, f_2, \ldots, f_n)^T$ is a vector of complex nonlinear functions, $\mathbf{F}(\mathbf{z})$ is an $n \times n$ complex matrix and the elements of it are functions of state complex variables, $\mathbf{A} = (a_1, a_2, \ldots, a_n)^T$ is an $n \times 1$ real (or complex) vector of system parameters and superscripts r and i stand for the real and imaginary parts of the state complex vector $\mathbf{z}$, respectively. On the other hand, the response chaotic complex system is assumed as

$$\dot{\mathbf{w}} = \mathbf{G}(\mathbf{w})\mathbf{B} + \mathbf{g}(\mathbf{w}) + \mathbf{v}, \tag{2}$$

where $\mathbf{w} = [w_1, w_2, \ldots, w_n]^T$ is a state complex vector, and $\mathbf{w} = \mathbf{w}^r + j\mathbf{w}^i, \mathbf{w}^r = (u'_1, u'_3, \ldots, u'_{2n-1}), \mathbf{w}^i = (u'_2, u'_4, \ldots, u'_{2n})^T$. $\mathbf{g} = (g_1, g_2, \ldots, g_n)^T$ is a vector of complex nonlinear functions, $\mathbf{G}(\mathbf{w})$ is an $n \times n$ complex matrix and its elements are functions of state complex variables, $\mathbf{B} = (b_1, b_2, \ldots, b_n)^T$ is an $n \times 1$ real (or complex) vector of system parameters and the designed controller is $\mathbf{v} = \mathbf{v}^r + j\mathbf{v}^i$, where $\mathbf{v}^r = (v_1, v_3, \ldots, v_{2n-1})$ and $\mathbf{v}^i = (v_2, v_4, \ldots, v_{2n})$.

Define the anti-synchronization error vector as $\mathbf{e}(t) = \mathbf{z}(t) + \mathbf{w}(t)$, where $\mathbf{e}^r(t) = \mathbf{z}^r(t) + \mathbf{w}^r(t)$ and $\mathbf{e}^i(t) = \mathbf{z}^i(t) + \mathbf{w}^i(t)$, and $\mathbf{e}^r(t) = (e_1(t), e_3(t), \ldots, e_{2n-1}(t))^T, \mathbf{e}^i(t) = (e_2(t), e_4(t), \ldots, e_{2n}(t))^T$. The essential goal is to design controller $\mathbf{v}$ such that anti-synchronization error vector $\mathbf{e}(t)$ tends to zero as $t \to \infty$, namely,

$$\begin{aligned}
\lim_{t\to\infty} \|\mathbf{e}^r(t)\| &= \lim_{t\to\infty} \|\mathbf{z}^r(t, \mathbf{z}^r_{\mathbf{0}}) + \mathbf{w}^r(t, \mathbf{w}^r_{\mathbf{0}})\| \\
&= \lim_{t\to\infty} \sum_{k=1}^{n} \|u_{2k-1}(t, u_{2k-1}(0)) + u'_{2k-1}(t, u'_{2k-1}(0))\| \\
&= 0, \\
\lim_{t\to\infty} \|\mathbf{e}^i(t)\| &= \lim_{t\to\infty} \|\mathbf{z}^i(t, \mathbf{z}^i_{\mathbf{0}}) + \mathbf{w}^i(t, \mathbf{w}^i_{\mathbf{0}})\| \\
&= \lim_{t\to\infty} \sum_{k=1}^{n} \|u_{2k}(t, u_{2k}(0)) + u'_{2k}(t,\ u'_{2k}(0))\| \\
&= 0,
\end{aligned}$$

then

$$\lim_{t\to\infty} \|\mathbf{e}(t)\| = \lim_{t\to\infty} \|\mathbf{e}^r(t)\| + \lim_{t\to\infty} \|\mathbf{e}^i(t)\| = 0,$$

where $\mathbf{z}^r_{\mathbf{0}} = [u_1(0), u_3(0), \ldots, u_{2n-1}(0)]^T$ and $\mathbf{z}^i_{\mathbf{0}} = [u_2(0), u_4(0), \ldots, u_{2n}(0)]^T$ are initial values of real and imaginary parts of the complex system (1), while $\mathbf{w}^r_{\mathbf{0}} = [u'_1(0), u'_3(0), \ldots, u'_{2n-1}(0)]^T$ and $\mathbf{w}^i_{\mathbf{0}} = [u'_2(0), u'_4(0), \ldots, u'_{2n}(0)]^T$ are one of the complex system (2), so the initial values of real and imaginary parts of error system are $\mathbf{e}^r_{\mathbf{0}} = [u_1(0) + u'_1(0), u_3(0) + u'_3(0), \ldots, u_{2n-1}(0) + u'_{2n-1}(0)]^T$ and $\mathbf{e}^i_{\mathbf{0}} = [u_2(0) + u'_2(0), u_4(0) + u'_4(0), \ldots, u_{2n}(0) + u'_{2n}(0)]^T$; $\|\cdot\|$ denotes the Euclidean norm.

3. Adaptive anti-synchronization controller design

Theorem 1. *If nonlinear controller is designed as*

$$\begin{aligned}
\mathbf{v} &= \mathbf{v}^r + j\mathbf{v}^i = -\mathbf{F}(\mathbf{z})\hat{\mathbf{A}} - \mathbf{f}(\mathbf{z}) - \mathbf{G}(\mathbf{w})\hat{\mathbf{B}} - \mathbf{g}(\mathbf{w}) - k\mathbf{e} \\
&= -\mathbf{F}^r(u_1, u_3, \ldots, u_{2n-1})\hat{\mathbf{A}} - \mathbf{f}^r(u_1, u_2, \ldots, u_n) - \mathbf{G}^r(u'_1, u'_3, \ldots, u'_{2n-1})\hat{\mathbf{B}} - \mathbf{g}^r(u'_1, u'_2, \ldots, u'_n) - k\mathbf{e}^r \\
&\quad + j[-\mathbf{F}^i(u_2, u_4, \ldots, u_{2n})\hat{\mathbf{A}} - \mathbf{f}^i(u_1, u_2, \ldots, u_n) - \mathbf{G}^i(u'_2, u'_4, \ldots, u'_{2n})\hat{\mathbf{B}} - \mathbf{g}^i(u'_1, u'_2, \ldots, u'_n) - k\mathbf{e}^i]
\end{aligned} \tag{3}$$

and adaptive laws of parameters are selected as

$$\begin{cases}
\dot{\hat{\mathbf{A}}} = [(\mathbf{F}^r(u_1, u_3, \ldots, u_{2n-1}))^T, (\mathbf{F}^i(u_2, u_4, \ldots, u_{2n}))^T] \begin{bmatrix} \mathbf{e}^r \\ \mathbf{e}^i \end{bmatrix}, \\
\dot{\hat{\mathbf{B}}} = [(\mathbf{G}^r(u'_1, u'_3, \ldots, u'_{2n-1}))^T, (\mathbf{G}^i(u'_2, u'_4, \ldots, u'_{2n}))^T] \begin{bmatrix} \mathbf{e}^r \\ \mathbf{e}^i \end{bmatrix},
\end{cases} \tag{4}$$