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On second order duality for minimax fractional programming☆

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ABSTRACT

In this paper, two types of second order duality for minimax fractional programming are formulated by introducing an additional vector r. The weak, strong and converse duality theorems are proved for these programs under η-bonvexity assumptions. Several results including many recent works are obtained as special cases.

1. Introduction

In this paper, we consider the following minimax fractional programming problem:

$$\text{(P)}\quad \text{Minimize}\quad \psi(x)=\sup_{y\in Y}\frac{f(x,y)}{h(x,y)}$$
$$\text{s.t.}\quad g(x)\le 0,\ x\in R^n,$$

where Y is a compact subset of R^l, $f(.,.) : R^n \times R^l \to R$, $h(.,.) : R^n \times R^l \to R$ are twice continuously differentiable on $R^n \times R^l$ and $g(.,.) : R^n \to R^m$ is twice continuously differentiable on R^n. It is assumed that for each (x, y) in $R^n \times R^l$, $f(x, y) \ge 0$ and $h(x, y) \ge 0$.

Since minimax fractional programming has wide applications (see [1–4]), much attention has been paid to optimality conditions and duality theorems for minimax fractional programming problems. For the case of convex differentiable minimax fractional programming, Yadav and Mukherjee [5] formulated two dual models for (P) and derived duality theorems. Chandra and Kumar [6] pointed out certain omissions in the dual formulation of Yadav and Mukherjee; they constructed two modified dual problems for minimax fractional programming problem and proved duality results. Liu and Wu [7,8], and Ahmad [9] obtained sufficient optimality conditions and duality theorems for (P) assuming the functions involved to be generalized convex. Yang and Hou [10] discussed optimality conditions and duality results for (P) involving generalized convexity assumptions.

Mangasarian [11] introduced the notation of second order duality for nonlinear programs by introducing an additional vector $p \in R^n$. He has indicated a possible computational advantage of the second order dual over the first order dual. Instead of imposing explicit condition on p, Mond [12] included p in a second order type convexity. Bector et al. [13] discussed second order duality results for minimax programming problems under generalized B-invexity. Later on, Liu [14] extended these

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results involving second order generalized B-invexity. Recently, Husain et al. [15] have formulated two types of second order dual models for minimax fractional programming problems, and derived weak, strong and strict converse duality theorems under η-bonvexity assumptions.

In this paper, two types of second order duality in minimax fractional programming are formulated by introducing an additional vector r. The weak, strong and converse duality theorems are proved for these programs under η-bonvexity assumptions. Our results generalize these existing dual formulations which were discussed by the authors in [9,13,6,14,7,8,16,5,10,15].

2. Preliminaries

Let $S = \{x \in R^n : g(x) \leq 0\}$ denote the set of all feasible solutions of (P). For each $(x, y) \in R^n \times R^l$, we define

$$J(x) = \{j \in M = \{1, 2, \ldots, m\} : g_j(x) = 0\},$$

$$Y(x) = \{y \in Y : f(x, y) = \sup_{z \in Y} f(x, z)\},$$

and

$$K(x) = \left\{(s, t, \widetilde{y}) \in N \times R^s_+ \times R^{ls} : 1 \leq s \leq n+1, t = (t_1, t_2, \ldots, t_s) \in R^s_+, \right.$$
$$\left. \sum_{t=1}^{s} t_i = 1, \widetilde{y} = (\widetilde{y}_1, \widetilde{y}_2, \ldots, \widetilde{y}_s), \widetilde{y}_i \in Y(x), i = 1, 2, \ldots, s \right\}.$$

Let $f : R^n \to R$ be a twice differentiable function.

Definition 2.1 (*[17]*)**.** Function f is said to be η-bonvex at $\bar{x} \in R^n$, if there exists a certain mapping $\eta : R^n \times R^n \to R^n$ such that for all $x, p \in R^n$, we have

$$f(x) - f(\bar{x}) + \frac{1}{2}p^T \nabla^2 f(\bar{x}) p \geq \eta^T(x, \bar{x})[\nabla f(\bar{x}) + \nabla^2 f(\bar{x}) p].$$

Definition 2.2 (*[17]*)**.** Function f is said to be strictly η-bonvex at $\bar{x} \in R^n$, if there exists a certain mapping $\eta : R^n \times R^n \to R^n$ such that for all $x, p \in R^n$, we have

$$f(x) - f(\bar{x}) + \frac{1}{2}p^T \nabla^2 f(\bar{x}) p > \eta^T(x, \bar{x})[\nabla f(\bar{x}) + \nabla^2 f(\bar{x}) p].$$

The following theorem will be needed in the proofs of strong duality theorems:

Theorem 2.1 (*Necessary Conditions [6]*)**.** *Let x^* be a solution of* (P) *and let $\nabla g_j(x^*), j \in J(x^*)$ be linearly independent. There exist $(s^*, t^*, \bar{y}^*) \in K(x^*)$, $\lambda^* \in R_+$ and $\mu^* \in R^m_+$ such that*

$$\nabla \sum_{i=1}^{s^*} t_i^*(f(x^*, \bar{y}_i^*) - \lambda^* h(x^*, \bar{y}_i^*)) + \nabla \sum_{j=1}^{m} \mu_j^* g_j(x^*) = 0,$$

$$f(x^*, \bar{y}_i^*) - \lambda^* h(x^*, \bar{y}_i^*) = 0, \quad i = 1, 2, \ldots, s^*,$$

$$\sum_{j=1}^{m} \mu_j^* g_j(x^*) = 0,$$

$$t_i^* \geq 0, \quad \sum_{i=1}^{s^*} t_i^* = 1, \qquad \bar{y}_i^* \in Y(x^*), \quad i = 1, 2, \ldots, s^*.$$

3. First duality model

By utilizing the necessary optimality conditions of the previous section, we formulate the following second order dual to (P) as follows:

$$\text{(MD)} \qquad \max_{(s,t,\bar{y}) \in K(z)} \; \sup_{(z,\mu,\lambda,r,p) \in H_1(s,t,\bar{y})} \; \lambda,$$