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# An analysis of the swimming problem of a singly flagellated micro-organism in an MHD fluid flowing through a porous medium

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ABSTRACT

The focus of the present work is concerned with the study of the swimming of microscopic organisms that use a single flagellum for propulsion in a magnetohydrodynamic (MHD) fluid flowing through a porous medium. The flow is modelled by appropriate equations and the organism is modelled by an infinite flexible but inextensible transversely waving sheet, which represents approximately the flagellum. The governing equations subject to appropriate boundary conditions are solved analytically. Expressions for the velocity of propulsion of the microscopic organism are obtained. We show that as the MHD character of the fluid is removed the results match those of an earlier analysed problem of propulsion through a fluid in a porous medium. In addition, in the final case of a simple viscous fluid (absence of magnetic field), we show that as the permeability becomes large the results reduce to the swimming of such organisms in a viscous fluid (discounting the pores and the MHD character).



## 1. Introduction

The propulsion of microscopic organisms where a single Flagellum is employed for propulsion is a well-known mechanism of motion. Several micro-organisms are known to use a single flagellum for propulsion, for instance algae, spermatozoa and certain bacteria cf. [1,2]. The problem of the swimming of microscopic organisms in a viscous fluid is well documented; see for instance, [3–10]. Further to this the swimming of microscopic organisms in a porous medium has been studied by [11] and from a mathematical point of view by [12].

Fluids that are under the influence of a magnetic field, are referred to as magnetohydrodynamic (MHD) fluids; we consider the flow of such a fluid through a porous medium. With this fact in mind the approach here is to consider the swimming of singly flagellated micro-organisms in a MHD fluid flowing through a porous medium. Clearly the dynamics of the problem is very complicated as such we will focus on models for single-phase flow through porous media. The application of Newtonian fluid models to porous media requires a well-defined description of the porous matrix boundaries, which is a difficult task. However, in order to get a macroscopic viewpoint of what happens when a microscopic organism swims through this complex porous structure we have to resort to averaged models that represent the structure. Previously, [12] analysed the problem of a microscopic organism swimming through a viscous fluid flowing through a porous medium. We will alter this model to consider the MHD fluid flowing through the porous medium. For purposes of generality we assume that $\mu_{eff} \neq \mu$.

The microscopic organism is assumed to be propelled by a single flagellum, which is modelled by a doubly infinite sheet, flexible but inextensible (cf. [3,6]) where motion is induced by small transverse oscillations. The solutions of the equations

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**List of mathematical symbols**

| | |
|---|---|
| c | $= \sigma/k$, phase velocity of the wave representing the micro-organism |
| $l$ | $= \left(k^2 + \frac{i\sigma}{\nu}\right)^{1/2}$ |
| $l_p$ | $= \left(k^2 + \frac{i\sigma}{\nu_{eff}}\right)^{1/2}$ |
| $m$ | $= \frac{\nu}{\nu_{eff}\,\kappa}$ |
| p | interstitial pressure |
| $u$ | averaged velocity in the $x$ direction |
| $v$ | averaged velocity in the $y$ direction |
| $\gamma_p$ | $= 2\mathrm{Re}\left[(l_p^2 + m)^{1/2}\right]$ |
| $\gamma_{p'}$ | $= 2\mathrm{Re}\left[(l_p^2 + \kappa^{-1})^{1/2}\right]$ |
| $\kappa$ | permeability |
| $\mu$ | viscosity |
| $\mu_{eff}$ | effective viscosity of the fluid in the medium |
| $\nu$ | $= \mu/\rho$ |
| $\nu_{eff}$ | $= \mu_{eff}/\rho$ |
| $\rho$ | fluid density |
| $\psi$ | Two-dimensional stream function |
| $\omega$ | $= -\nabla^2\psi$ |

are presented and expressions for the velocity of propulsion are developed. An allied case showing that if the magnetic field strength is reduced to zero the solutions revert to the case examined by [12] of propulsion in a viscous fluid flowing through a porous medium. In addition, by further removing the porous structure, i.e., in the case of large permeability the results match the results of propulsion in a hydrodynamic viscous fluid (cf. [5]); these cases are presented as a mathematical verification of the results.

## 2. Equations of motion and formulation of the problem

A magnetic field of constant strength $H_o$ is applied in a direction perpendicular to the flow of the fluid. The equations governing the flow of an incompressible viscous fluid through a porous medium under the influence of a transverse magnetic field are given by

$$\mu_{eff}\,\nabla^2 u - \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \left(\frac{\mu}{\kappa} + \sigma_m B_o^2\right)u, \tag{1}$$

$$\mu_{eff}\,\nabla^2 v - \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \frac{\partial p}{\partial y} + \frac{\mu}{\kappa}v, \tag{2}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

where $u$ and $v$ are the averaged velocities in the $x$ and $y$ directions respectively, $p$ is the interstitial pressure, $\rho$ is density, $\sigma_m$ is the electrical conductivity of the fluid, and $B_o = \mu_o H_o$, $\mu_o$ being the magnetic permeability. Here, the quantities $\rho$ and $\sigma_m$ are constants throughout the flow field. The total magnetic field $B$ is perpendicular to the velocity $V$ and induced magnetic field is negligible compared with the imposed magnetic field so that the magnetic Reynolds number is small. The electric field is assumed zero. In addition, $\kappa$ is the permeability, $\mu$ is the coefficient of viscosity and $\mu_{eff}$ is the effective viscosity of the fluid in the medium (which is in general different from the viscosity of the fluid $\mu$). The analysis here will adopt the condition that $\mu_{eff} \neq \mu$. The following expression gives a relationship between the two viscosities [13]

$$\mu_{eff} = \mu\left\{1 + \frac{5\beta}{2} + \left(\frac{81}{32} - \log\beta + 19.66\right)\beta^2 + 6.59\beta^{5/2}\log\beta\right\}$$

where $\beta = 1 - \eta = 4\pi d_p^3$, and $d_p$ is the average pore diameter. We will discuss the case with $\mu_{eff} = \mu$ later.

The analysis here will be concerned with two-dimensions only. With this in mind we proceed with our analysis by defining a two-dimensional stream function $\psi(x, y)$ such that

$$u = \psi_y \quad \text{and} \quad v = -\psi_x,$$