



Periodic solutions for a class of reaction–diffusion equations with p -Laplacian

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ABSTRACT

In this paper we study non-trivial, non-negative periodic solutions of certain periodic reaction–diffusion equations with the p -Laplacian under the homogeneous Dirichlet boundary condition. First, we prove the existence of such periodic solutions, and provide a priori estimates for their upper bound using Moser iteration. We also show that the support of these solutions is independent of time. Further, we establish the attractivity of maximal periodic solutions using the monotonicity method. One of our motivations is a generalized Verhulst model with time-periodicity and nonlinear diffusion in a bounded heterogeneous environment.

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1. Introduction

In population dynamics, the classical logistic growth model, or Verhulst model, takes the form $u' = mu(1 - bu)$. In this model, it is assumed that the dynamics is independent of spatial location (a more realistic interpretation is that only the average population density is studied), and the growth and self-limitation (competition) of the species are constant (independent of time). In the last few decades, many modifications to the model have been introduced, and increasingly sophisticated models have been studied. For example, the diffusive model $u_t - \Delta u = u(m - bu)$ takes into account spatial dependence, and assumes that population diffuses to areas of lower concentration through random movement. Further, the heterogeneous environment model takes into account spatial dependence of the growth rate and competition rate by taking m and b to be functions of x .

In this paper, we further generalize the Verhulst model by introducing the following new elements:

(i) Nonlinear diffusion: We assume that, instead of random movement, population diffuses to areas of lower concentration according to the gradient in concentration; in other words, we assume that the diffusion is gradient-driven. Specifically, we shall model the diffusion by the p -Laplacian instead of the standard Laplacian.

(ii) Time-periodicity: In addition to the spatial dependence of the heterogeneous model, we also take into account the natural assumption that growth and competition rates are seasonal; in other words, we assume that m and b are functions of x and t , and that they are T -periodic in t .

(iii) Finally, we generalize the model by assuming that growth and competition may depend nonlinearly on the population density. This introduces the exponents α and β in our model below.

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The above considerations thus lead us to the following periodic reaction–diffusion problem with p -Laplacian:

$$\frac{\partial u}{\partial t} - \operatorname{div}(|\nabla u|^{p-2} \nabla u) = u^\alpha (m(x, t) - b(x, t)u^\beta), \quad (x, t) \in \Omega \times \mathbb{R}^+, \quad (1.1)$$

$$u(x, t) = 0, \quad (x, t) \in \partial\Omega \times \mathbb{R}^+, \quad (1.2)$$

$$u(x, 0) = u_0(x), \quad x \in \Omega, \quad (1.3)$$

where $p > 2$, $1 \leq \alpha < p - 1$, $\beta > 0$, the functions m and b are continuous and T -periodic, and Ω is a bounded domain of \mathbb{R}^N with smooth boundary $\partial\Omega$.

In this population dynamics model, the species is restricted to the bounded heterogeneous environment Ω whose boundary is prohibitive to the species. We are interested in nonnegative solutions to the problem.

This equation is in fact much more widely applicable than in population dynamics. Nonlinear p -Laplacian equations appear in many applications such as non-Newtonian fluid flow, gas flow in porous media and population dynamics. The reaction term of the Eq. (1.1) consists of the source term $m(x, t)u^\alpha$ and absorption term $b(x, t)u^{\alpha+\beta}$. Reaction–diffusion equations with such reaction terms can be regarded as generalizations of Fisher or Kolomogorv–Petrovsky–Piscunov equations.

Many papers are concerned with the steady-state or elliptic version of the problem (1.1) and (1.2) when the functions m and b are autonomous, i.e., independent of t ; see for example [1–4] and references therein. In the past two decades, there is also considerable interest in the existence, uniqueness, regularity and other qualitative properties, such as blow-up and finite speed of propagation of disturbances, of weak solutions of the initial-boundary value problem; see for example [5–9].

When $p = 2$, (1.1) becomes a semi-linear periodic parabolic equation, and the problem has been studied by many researchers [10–17]. The technique used for the semi-linear problem relies heavily on the associated linearized periodic eigenvalue problem [12], and is clearly not applicable for $p \neq 2$. In fact, although the eigenvalue problem for the p -Laplacian elliptic operator under the homogeneous Dirichlet boundary condition has been investigated extensively [4, 18–20], little is known about the periodic parabolic eigenvalue problem with p -Laplacian ($p \neq 2$) as yet. Most of the known results so far concern the existence of periodic solutions [21–25].

The main result of this paper is the existence of maximal periodic solutions of (1.1) and (1.2) and their attractivity with respect to solutions of the initial-boundary value problem (1.1)–(1.3). The paper is organized as follows: After some preliminaries in Section 2, we give the existence result for the periodic problem (1.1) and (1.2) in Section 3. This is accomplished by extending the monotone iterative method for regular parabolic boundary value problems to the periodic degenerate parabolic problem at hand. It should be noted that the monotone iterative method used here bypasses the necessity to find lower bounds (as in [22, 23], for example) which can put undue restrictions on the reaction terms. In Section 4 we provide a priori estimates for the upper bound of these periodic solutions using Moser iteration. These a priori estimates are crucial for the proof of the attractivity of the maximal periodic solution using the monotonicity method, which is presented in Section 5. To round off Section 4, we also prove that the support of periodic solutions is independent of t . Regarding this last result, the biological interpretation is that the population does not vanish in any part of the domain at any time.

2. Preliminaries

The notations used in this paper are standard. For example, $\|\cdot\|_p$ and $\|\cdot\|_{1,p}$ ($1 \leq p \leq \infty$) refer to L^p and $W^{1,p}$ norms respectively. Unless otherwise stated, the reference to the space Ω is understood and will not be mentioned explicitly.

Throughout this paper, we assume that

(H1) $b(x, t) \in C_T(\overline{Q_T})$ is non-negative, where $Q_T = \Omega \times (0, T)$ and $C_T(\overline{Q_T})$ is the class of functions that are continuous in $\overline{\Omega} \times \mathbb{R}$ and T -periodic with respect to t .

(H2) $m(x, t) \in C_T(\overline{Q_T})$ may change sign, but

$$(a) \quad \Omega^+ := \left\{ x \in \Omega : \frac{1}{T} \int_0^T m(x, t) dt > 0 \right\} \neq \emptyset$$

or

$$(b) \quad \Omega^+ := \{x \in \Omega : m(x, t) > 0 \text{ for } t \in [0, T]\} \neq \emptyset.$$

By (H2a), there exists $x_0 \in \Omega$ such that $\frac{1}{T} \int_0^T m(x_0, t) dt = m_0 > 0$. Moreover by the continuity of $m(x, t)$, there exists $\delta > 0$ such that

$$\frac{1}{T} \int_0^T \min_{x \in B(x_0, \delta)} m(x, t) dt \geq \frac{m_0}{2}. \quad (2.1)$$

Let λ_1 and $\psi_1(x)$, respectively, be the first eigenvalue and its corresponding eigenfunction of the eigenvalue problem

$$\begin{aligned} -\operatorname{div}(|\nabla \psi|^{p-2} \nabla \psi) &= \lambda |\psi|^{p-2} \psi && \text{in } B(x_0, \delta) \\ \psi &= 0 && \text{on } \partial B(x_0, \delta). \end{aligned}$$

By [19], we know that $\lambda_1 > 0$, and that $\psi_1(x)$ is positive in $B(x_0, \delta)$, belongs to $C^{1,\nu}(\overline{B(x_0, \delta)})$ for some $\nu \in (0, 1)$ and is of class $C^{2,\nu}$ in a neighborhood of $\partial B(x_0, \delta)$.

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