Nonlinear Analysis: Real World Applications 11 (2010) 562–573

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Exponential convergence of BAM neural networks with time-varying coefficients and distributed delays

Ranchao Wu *

School of Mathematics, Anhui University, Hefei 230039, China

ARTICLE INFO

Article history:
Received 20 October 2007
Accepted 5 February 2009

Keywords:
BAM neural networks
Time-varying coefficient
Distributed time delay
Exponential convergence
Periodic solution
Coincidence degree

ABSTRACT

In the current paper, BAM neural networks with time-varying coefficients and distributed time delays are studied. Sufficient conditions guaranteeing the exponential componentwise convergence and existence of one unique periodic solution are obtained by the comparison principle, continuation theorem of topological degree and inequality techniques. The boundedness and differentiability of activation functions are removed. The obtained sufficient criteria are easy to verify and are hence very useful in applications.

1. Introduction

Bidirectional associative memory (BAM) neural model, proposed by Kosko [1–3], is a two-layer nonlinear feedback network model. It is often known as an extension of the unidirectional autoassociator of the Hopfield model [4], generalizing the single-layer autoassociative Hebbian correlation to a two-layer pattern-matched heteroassociative circuit. Since the BAM model presents a flexible nonlinear mapping from input space to output one, it has promising potential for applications in pattern recognition and automatic control, and has been extensively studied over the past years, see, for example, Refs. [5–19].

As we know, practical applications heavily depend on the dynamical behavior, especially the asymptotic stability, of neural networks. So the stability of neural networks has been one of the most active areas of research, and has been widely investigated, see, e.g., [5–28]. Due to the finite transmission speed of signals among neurons, time delays, whether constant or variable, are ubiquitous in neural networks, and will cause instability, divergence and oscillations in the models [29]. Therefore, the asymptotic stability of delayed BAM models has been investigated by many researchers and various sufficient conditions have been given. On the other hand, neural networks usually have a spatial nature due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, the distribution of propagation is not instantaneous and cannot be modeled by constant time delays. As a result, continuously distributed delays should be incorporated into the BAM neural models so that the distant past has less influence compared to the recent behavior of the state. So BAM neural networks with distributed delays are also of great theoretical and practical importance. There exist some results on such research [10,11,14–19].

The existing literature on BAM neural networks is overwhelmingly concerned with autonomous systems with constant parameters and external stimuli. However, the neuron charging time, interconnection weights and external inputs often change as time proceeds. It is necessary that BAM neural models with temporal structure of neural activities should be introduced and investigated. One can refer to Refs. [7,8,10,15,18] for discussions in this respect.

* Fax: +86 551 5106731.
E-mail addresses: wuranchao@yahoo.com.cn, rcwu@ahu.edu.cn.

1468-1218/$ – see front matter © 2009 Published by Elsevier Ltd
doi:10.1016/j.nonrwa.2009.02.003

Now BAM neural networks with time-varying coefficients and distributed delays are further considered in this paper. Employing the comparison principle of delayed differential equations, the continuation theorem of topological degree and analytic techniques such as Hölder and Minkowski inequalities, we obtain several sufficient conditions ensuring the exponential convergence of this model, the existence and stability of one unique periodic solution, with no restrictions of boundedness or differentiability imposed on activation functions. Furthermore, the exponential convergence introduced here is in componentwise sense, more stronger than conventional Lyapunov exponential stability. An illustrative example is also given to show the effectiveness of the results.

The paper is organized as follows. In Section 2, a BAM neural model is formulated and some assumptions are given. Section 3 is devoted to the exponential convergence of this model. In Section 4, the existence of a unique periodic solution is discussed when the neural model is periodic.

2. Preliminaries

Consider the following BAM neural networks with time-varying coefficients and distributed delays:

$$\frac{\mathrm{d}u_i}{\mathrm{d}t} = -a_i(t)e_i(u_i) + \sum_{j=1}^{n} b_{ji}(t)f_j(v_j) + \sum_{j=1}^{n} l_{ji}(t)\int_0^{\tau} k_{ji}(s)g_j(v_j(t-\tau_{ji}-s))\mathrm{d}s + I_i(t), \tag{1}$$

$$\frac{\mathrm{d}v_j}{\mathrm{d}t} = -c_j(t)h_j(v_j) + \sum_{i=1}^{m} d_{ij}(t)p_i(u_i) + \sum_{i=1}^{m} \tilde{l}_{ij}(t)\int_0^{\sigma} \tilde{k}_{ij}(s)q_i(u_i(t-\sigma_{ij}-s))\mathrm{d}s + J_j(t), \tag{2}$$

with initial values

$$\begin{aligned} &u_i(s) = \phi_{ui}(s), \quad -h \le s \le 0,\\ &v_j(s) = \phi_{vj}(s), \quad -\tilde{h} \le s \le 0, \end{aligned} \tag{3}$$

where $i = 1, \ldots, m, j = 1, \ldots, n, t \in R^+ = [0, +\infty), \tau > 0, \sigma > 0,$

$$h = \sigma + \sigma^* \quad \sigma^* = \max_{1\le i\le m, 1\le j\le n}\{\sigma_{ij}\},$$

$$\tilde{h} = \tau + \tau^*, \quad \tau^* = \max_{1\le i\le m, 1\le j\le n}\{\tau_{ji}\};$$

$u = \mathrm{col}(u_1, \ldots, u_m) \in R^m$, $v = \mathrm{col}(v_1, \ldots, v_n) \in R^n$, $u_i(t)$ and $v_j(t)$ are the states of the ith neuron and the jth neuron at time t, respectively; continuous functions $a_i(t)$ and $c_j(t)$ represent the neuron charging times, $b_{ji}(t)$, $l_{ji}(t)$, $d_{ij}(t)$ and $\tilde{l}_{ij}(t)$ are the weights of the neuron interconnections; τ_{ji} and σ_{ij} represent the axonal signal transmission delays; g_j, f_j, p_i and q_i denote the activation functions of the neurons; continuous functions $I_i(t)$ and $J_j(t)$ are the external inputs on the neurons; the initial value functions $\phi_{ui}(s)$ and $\phi_{vj}(s)$ are bounded and continuous on $[-h, 0]$ and $[-\tilde{h}, 0]$, respectively.

To investigate the exponential convergence of Eqs. (1) and (2), we make further assumptions ($i = 1, \ldots, m, j = 1, \ldots, n$):

(H1) Suppose that $a_i(t), c_j(t), b_{ji}(t), d_{ij}(t), l_{ji}(t), \tilde{l}_{ji}(t), I_i(t)$ and $J_j(t)$ are bounded, continuous functions defined on R^+, moreover, $a_i(t) > 0, c_j(t) > 0$;

(H2) Assume that functions $e_i, h_j : R \to R$ satisfy

$$(\xi-\eta)(e_i(\xi)-e_i(\eta)) \ge E_i(\xi-\eta)^2,$$
$$(\xi-\eta)(h_j(\xi)-h_j(\eta)) \ge H_j(\xi-\eta)^2,$$

with $e_i(0) = 0, h_j(0) = 0$, constants $E_i > 0, H_j > 0$, and there exist positive constants F_j, G_j, P_i and Q_i, such that f_j, g_j, p_i and q_i satisfy

$$0 \le \frac{f_j(\xi)-f_j(\eta)}{\xi-\eta} \le F_j, \qquad 0 \le \frac{g_j(\xi)-g_j(\eta)}{\xi-\eta} \le G_j,$$

$$0 \le \frac{p_i(\xi)-p_i(\eta)}{\xi-\eta} \le P_i, \qquad 0 \le \frac{q_i(\xi)-q_i(\eta)}{\xi-\eta} \le Q_i,$$

with $f_j(0) = g_j(0) = 0, p_i(0) = q_i(0) = 0$, for any $\xi, \eta \in R$;

(H3) Assume that $k_{ji}(t)$ and $\tilde{k}_{ij}(t)$ are positive, piecewise continuous and satisfy

$$\int_0^{\tau} \mathrm{e}^{\theta t}k_{ij}(t)\mathrm{d}t = \rho(\theta, \tau), \qquad \int_0^{\sigma} \mathrm{e}^{\theta t}\tilde{k}_{ji}(t)\mathrm{d}t = \tilde{\rho}(\theta, \sigma),$$

where $\rho(\theta, \tau)$ and $\tilde{\rho}(\theta, \sigma)$ are continuous in θ. When $\tau = \infty, \sigma = \infty$, $\rho(\theta) \equiv \rho(\theta, \tau)$ and $\tilde{\rho}(\theta) \equiv \tilde{\rho}(\theta, \sigma)$, with $\rho(0) = 1, \tilde{\rho}(0) = 1$.