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Short communication

New exact solutions corresponding to the second problem of Stokes for second grade fluids

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ABSTRACT

New exact solutions for the velocity field corresponding to the second problem of Stokes, for second grade fluids, have been established by the Laplace transform method. These solutions, presented as a sum of the steady-state and transient solutions, are in accordance with the previous solutions obtained by a different technique. The required time to reach the steady state is determined by graphical illustrations. This time decreases if the frequency of the velocity increases. The effects of the material parameters on the decay of the transients are also investigated by graphs.

1. Introduction

While the flow of a second grade fluid has been studied in much detail, new exact solutions are welcome for flows having technical relevance. The flows to be considered here correspond to some reasonable physical problems, namely the motions due to the oscillations of an infinite plate (Stokes' second problem). Such motions are not only of fundamental theoretical interest but they also occur in many applied problems [1].

The first exact solutions corresponding to this problem for non-Newtonian fluids, are those obtained by Rajagopal [2] for second grade fluids. His solutions, represent the steady-state velocity of the fluid in several states of motion. The starting solutions corresponding to the same problems have been recently established by Fetecau and Fetecau [3] using Fourier sine transforms and S. Asghar et al. [4] by means of the Laplace transforms. Their extension to Oldroyd-B fluids has been realized by Aksel et al. [5] also by means of the Fourier sine transform.

The purpose of this note is to present new exact solutions for the starting velocity corresponding to the second problem of Stokes for second grade fluids. Generally, the starting solutions are very important for those who need to eliminate the transients from their rheological measurements. The established solutions, unlike those obtained by Erdogan for Newtonian fluids [6] and Asghar et al. [4] for second grade fluids, are presented in simpler forms as a sum of steady-state and transient solutions. Furthermore, they are in accordance with the previous solutions obtained by a different technique. More exactly, the steady-state solutions are identical to those obtained by Rajagopal while the diagrams of the transient solutions, as in Figs. 1 and 2, are almost identical to those obtained in [3]. The required time to reach the steady state for the cosine and sine oscillations of the boundary has been determined by graphical illustrations. This time decreases if the frequency of the velocity of the boundary increases.

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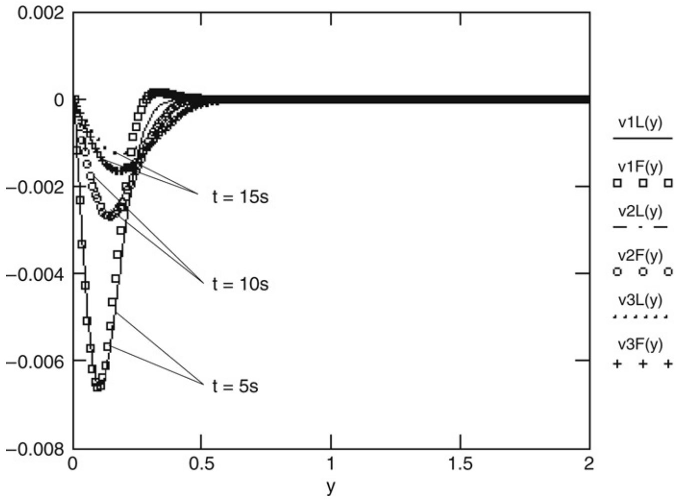

Fig. 1. Profiles of transient velocities $v_{tc}(y, t)$ given by Eq. (16) — curves $v1L(y)$, $v2L(y)$, $v3L(y)$ and Eq. (18) — curves $v1F(y)$, $v2F(y)$, $v3F(y)$, for cosine oscillations of the boundary (with an error of 10^{-4}), for $V = 1$, $\omega = 2$, $\beta = 11.746$, $\alpha = 0.0001$ and $\nu = 0.0011746$.

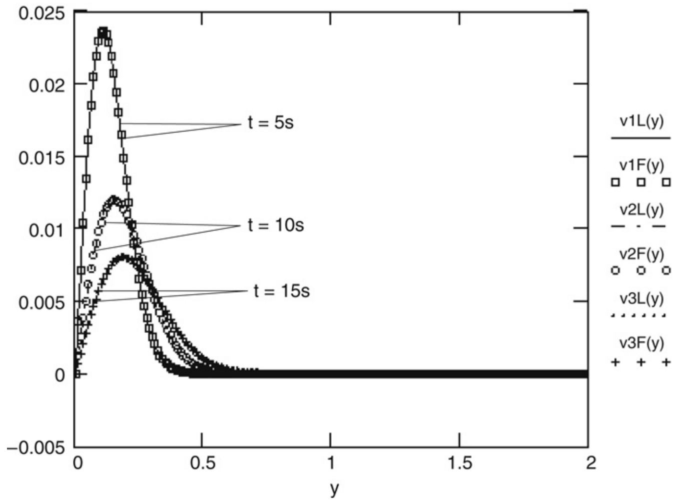

Fig. 2. Profiles of transient velocities $v_{ts}(y, t)$ given by Eq. (17) — curves $v1L(y)$, $v2L(y)$, $v3L(y)$ and Eq. (19) — curves $v1F(y)$, $v2F(y)$, $v3F(y)$, for sine oscillations of the boundary, with $V = 1$, $\omega = 2$, $\beta = 11.746$, $\alpha = 0.0001$ and $\nu = 0.0011746$.

2. Governing equation

Let us consider an incompressible second grade fluid at rest over an infinitely extended flat plate perpendicular to the y-axis of a Cartesian coordinate system x, y and z. At time $t = 0^+$ the flat plate begins to oscillate in its plane. Due to the shear the fluid above the plate is gradually moved, its velocity being of the form [2]

$$\mathbf{v} = \mathbf{v}(y, t) = u(y, t)\mathbf{i}, \tag{1}$$

where $\mathbf{i}$ is the unit vector along the x-direction. For such flows the constraint of incompressibility is automatically satisfied. In the absence of body forces and a pressure gradient in the flow direction, the governing equation is [2,3]

$$(\mu + \alpha_1 \partial_t)\partial_y^2 v(y, t) = \rho \partial_t v(y, t); \quad y, t > 0, \tag{2}$$

where μ and α_1 are material constants and ρ is the constant density of the fluid.

The boundary and initial conditions corresponding to the cosine and sine oscillations of the boundary are given by [3–5]

$$v(0, t) = V\cos(\omega t) \quad \text{or} \quad v(0, t) = V\sin(\omega t) \quad \text{for all } t > 0, \tag{3}$$

and

$$v(y, 0) = 0 \quad \text{for } y > 0, \tag{4}$$