Nonlinear Analysis: Real World Applications 13 (2012) 1353–1361

Contents lists available at SciVerse ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Exponential stability of reaction–diffusion high-order Markovian jump Hopfield neural networks with time-varying delays

Yangfan Wang [a,d,e], Ping Lin [a], Linshan Wang [b,c,*]

[a] Division of Mathematics, University of Dundee, Dundee, DD1 4HN, UK
[b] Department of Mathematics, Ocean University of China, Qingdao, 266071, China
[c] Department of Mathematics, Liaocheng University, Liaocheng, 252059, China
[d] Department of Mathematics and Mechanics, University of Science and Technology Beijing (USTB), Beijing, 100083, China
[e] College of Marine Life Sciences, Ocean University of China, Qingdao, 266071, China

ARTICLE INFO

Article history:
Received 20 September 2011
Accepted 28 October 2011

Keywords:
Reaction–diffusion high-order Hopfield neural networks
Time-varying delays
Markovian jump
Linear matrix inequalities
Exponential stability in the mean square

ABSTRACT

This paper studies the problems of global exponential stability of reaction–diffusion high-order Markovian jump Hopfield neural networks with time-varying delays. By employing a new Lyapunov–Krasovskii functional and linear matrix inequality, some criteria of global exponential stability in the mean square for the reaction–diffusion high-order neural networks are established, which are easily verifiable and have a wider adaptive. An example is also discussed to illustrate our results.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

Hopfield neural networks (HNNs) with time delays and their various generalization have been successfully employed in many areas such as pattern recognition, associate memory, and combinatorial optimization (see [1–9]). The earlier HNNs model proposed by Hopfield [10,11], based on analog circuit consisting of capacitors, resistors, and amplifiers, was represented by a system of ordinary equations.

Over the past few decades, the dynamics of reaction–diffusion Hopfield neural networks (RDHNNs) with time delays have been extensively investigated (see [12–17]), as regards the diffusion effect that cannot be avoided in the neural networks model when electrons are moving in an asymmetric electromagnetic field. So, the stability of RDHNNs with time delays should be a focused topic of theoretical as well as practical importance.

The systems with Markovian jumping parameters have attracted considerable attention in recent years (see, e.g., [18–22]). This is due to the fact that the systems sometimes have a phenomenon of information latching (see, [18]). A widely used approach to dealing with the information latching problem is to extract finite state representations. Thus, HNNs with Markovian jumping parameters have been discussed; see, for example, [18] and the references therein. In this paper, we study the problems of global exponential stability of reaction–diffusion high-order Markovian jump Hopfield neural networks (RDHOMJHNNs) with time-varying delays. This paper is also an extension of our previous work [23]. To the best of the authors' knowledge, there are very few results on stability of the equilibrium point for RDHOMJHNNs with time-varying delays in spite of the stability for the higher order HNNs and stochastic neural networks have been deeply

* Corresponding author at: Department of Mathematics, Ocean University of China, Qingdao, 266071, China.
E-mail address: wenxuetzz@gmail.com (L. Wang).

1468-1218/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.nonrwa.2011.10.013

studied (see, [18–29]). By employing a new Lyapunov–Krasovskii functional and linear matrix inequality (LMI), some criteria of global exponential stability in the mean square for the stochastic neural networks are established, which are easily verifiable. An example is also discussed to illustrate our results.

2. Model description and preliminaries

We consider the following RDHOMJHNNs with time-varying delays

$$\begin{cases} \dfrac{\partial u_i(t,x)}{\partial t} = \sum_{k=1}^{m} \dfrac{\partial}{\partial x_k}\left(D_{ik}(t,x,u)\dfrac{\partial u_i(t,x)}{\partial x_k}\right) - a_i(r(t))u_i(t,x) + \sum_{j=1}^{n} W_{ij}(r(t))f_j(u_j(t,x)) \\ \quad + \sum_{j=1}^{n} T_{ij}(r(t))g_j(u_j(t-\tau(t),x)) + \sum_{j=1}^{n}\sum_{l=1}^{n} T_{ijl}(r(t))g_j(u_j(t-\tau(t),x))g_l(u_l(t-\tau(t),x)) + V_i, \\ \dfrac{\partial u_i}{\partial n} := \mathrm{col}\left(\dfrac{\partial u_i}{\partial x_1},\ldots,\dfrac{\partial u_i}{\partial x_m}\right) = 0, \quad t \geq t_0 \geq 0,\ x \in \partial\Omega, \\ u_i(t_0+\theta,x) = \xi_i(\theta,x), \quad -\tau_0 \leq \theta \leq 0, \\ 0 \leq \tau(t) \leq \tau_0, \quad x \in \Omega,\ i,j = 1,\ldots,n \end{cases} \tag{1}$$

where $r(t),\ t > 0$ is a right-continuous Markov process on the probability space which takes values in the finite space $S = \{1, 2, \ldots, N\}$ with generator $G = \{\gamma_{ij}\}$ $(i, j \in S)$ (also called jumping transfer matrix) given by (see, [18])

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & \text{if } i \neq j \\ 1 + \gamma_{ii}\Delta + o(\Delta), & \text{if } i = j, \end{cases} \tag{2}$$

$\Delta > 0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$, and $\gamma_{ii} = -\sum_{j\neq i}\gamma_{ij}$.

Assume that $u(t,x) = (u_1(t,x),\ldots,u_n(t,x))^T \in R^n$, $x = (x_1,\ldots,x_m)^T \in R^m$, $a_i > 0$, $\tau(t)$, $D_{ik}(t,x,u) \geq 0$, W_{ij}, f_j, g_j, V_i, $\xi_i(\theta,x)$, $\partial u_i/\partial n = 0$ $(t \geq t_0 \geq 0, x \in \partial\Omega)$ and $\mu(\Omega) > 0$ have the same meanings as those in [17], T_{ij}, T_{ijl} are the first- and second-order synaptic weights of system (1) (see, [25]), and the superscript 'T' presents the transpose.

We assume throughout that the neuron activation functions $f_j(u_j), g_j(u_j),\ j = 1,\ldots,n$, satisfy the following conditions:

$$\begin{aligned} &(H_1):\ |g_j(u_j)| \leq M_j, \\ &0 \leq \frac{|f_j(u_j) - f_j(v_j)|}{|u_j - v_j|} \leq l_{0j}, \\ &0 \leq \frac{|g_j(u_j) - g_j(v_j)|}{|u_j - v_j|} \leq l_{1j}, \\ &\forall u_j \neq v_j, u_j, v_j, l_{0j}, l_{1j} \in R. \end{aligned} \tag{3}$$

If there is an equilibrium point $u^* = [u_1^*,\ldots,u_n^*]^T$ of system (1) with conditions (3), we can rewrite system (1) as the following equivalent form

$$\begin{cases} \dfrac{\partial(u_i(t,x) - u_i^*)}{\partial t} = \sum_{k=1}^{m} \dfrac{\partial}{\partial x_k}\left(D_{ik}(t,x,u)\dfrac{\partial u_i(t,x)}{\partial x_k}\right) - a_i(r(t))(u_i(t,x) - u_i^*) \\ \quad + \sum_{j=1}^{n} W_{ij}(r(t))(f_j(u_j(t,x)) - f_j(u_j^*)) \\ \quad + \sum_{j=1}^{n}\left(T_{ij}(r(t)) + \sum_{l=1}^{n}(T_{ijl}(r(t)) + T_{ilj}(r(t)))\zeta_l\right)(g_j(u_j(t-\tau(t),x)) - g_j(u_j^*)) \\ \dfrac{\partial u_i}{\partial n} := \mathrm{col}\left(\dfrac{\partial u_i}{\partial x_1},\ldots,\dfrac{\partial u_i}{\partial x_m}\right) = 0, \quad t \geq t_0 \geq 0,\ x \in \partial\Omega, \\ u_i(t_0+\theta,x) = \xi_i(\theta,x), \quad -\tau_0 \leq \theta \leq 0, \\ 0 \leq \tau(t) \leq \tau_0, \quad x \in \Omega,\ i = 1,\ldots,n, \end{cases} \tag{4}$$

where $\zeta_l = \frac{1}{2}(g_l(u_l(t-\tau(t),x)) + g_l(u_l^*))$ and $|\zeta_l| \leq M_l$.

Download English Version:

https://daneshyari.com/en/article/837781

Download Persian Version:

https://daneshyari.com/article/837781

Daneshyari.com