Nonlinear Analysis: Real World Applications 12 (2011) 1850–1864

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Permanence and almost periodic sequence solution for a discrete delay logistic equation with feedback control☆

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ARTICLE INFO

Article history:
Received 3 July 2010
Accepted 5 December 2010

Keywords:
Discrete
Delay logistic equation
Permanence
Almost periodic solution

ABSTRACT

In this paper, we consider a discrete delay logistic equation with feedback control. By applying the theory of difference inequality and constructing a suitable Lyapunov functional, sufficient conditions which guarantee the permanence and existence of a unique globally attractive positive almost periodic sequence solution of the system are obtained. The result of this paper is completely new. An example is employed to illustrate our main result.

1. Introduction

In this paper, we consider a discrete delay logistic equation with feedback control:

$$\begin{cases} x(n+1) = x(n)\exp\left\{r(n)\left[1 - \dfrac{x(n)}{K(n)} - \displaystyle\sum_{s_0=1}^{m_0} a_{s_0}(n)x(n-s_0)\right] - \displaystyle\sum_{s_1=0}^{m_1} b_{s_1}(n)u(n-s_1)\right\}, \\ u(n+1) = (1-\alpha(n))u(n) + \displaystyle\sum_{s_2=0}^{m_2} \beta_{s_2}(n)x(n-s_2), \end{cases} \tag{1}$$

where $x(n)$ is the density of the species at time n and $u(n)$ is the control variable at time n.

Denote as $\mathbf{R}$, $\mathbf{N}$, $\mathbf{Z}$ and $\mathbf{Z}^+$ the sets of real numbers, natural numbers, integers and nonnegative integers, respectively, $[a, b]_{\mathbf{Z}} = [a, b] \cap \mathbf{Z}$, $\forall a, b \in \mathbf{R}$.

Throughout this paper, we assume that

(H) $r(n), K(n), a_{s_0}(n), b_{s_1}(n), \alpha(n)$ and $\beta_{s_2}(n)$ are all bounded non-negative almost periodic sequences, $0 < \alpha^l \le \alpha(n) \le \alpha^u < 1$, $r^l > 0$, $s_0 = 1, 2, \ldots, m_0$, $s_1 = 0, 1, \ldots, m_1$, $s_2 = 0, 1, \ldots, m_2$.

Here, for any bounded sequence $\{a(n)\}$, $a^u = \sup_{n\in\mathbf{N}}\{a(n)\}$ and $a^l = \inf_{n\in\mathbf{N}}\{a(n)\}$.

Let $\tau = \max\{m_0, m_1, m_2\}$. We consider (1) together with the following initial conditions

$$\begin{aligned} &x(\theta) = \varphi(\theta) \ge 0, \quad \theta \in \{-\tau, -\tau+1, \ldots, 0\}, \ \varphi(0) > 0, \\ &u(\theta) = \psi(\theta) \ge 0, \quad \theta \in \{-\tau, -\tau+1, \ldots, 0\}, \ \psi(0) > 0. \end{aligned} \tag{2}$$

One can easily show that the solutions of (1) with initial condition (2) are defined and remain positive for all $n \in \mathbf{N}$.

☆ This work is supported by the National Natural Sciences Foundation of People's Republic of China under Grant 10971183.

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doi:10.1016/j.nonrwa.2010.12.001

A number of authors [1–3] have investigated the dynamical characteristics of the nonautonomous logistic differential equation

$$x'(t) = r(t)x(t)\left[1 - \frac{x(t)}{K(t)}\right]. \tag{3}$$

In many situations, $r(t)$ and $K(t)$ can be assumed to be nonconstant positive periodic functions with a common period ω to reflect the seasonal fluctuations. In such a periodic case, it has been shown that (3) has a positive ω-periodic solution $\tilde{x}(t)$ which attracts every positive solution $x(t)$ of (3) as $t \to +\infty$. See, e.g., [1–3].

Recently, many scholars have paid attention to the non-autonomous discrete population models, since the discrete time models governed by difference equation are more appropriate than the continuous ones when the populations have non-overlapping generations (see [4–10]). Moreover, since the discrete time models can also provide efficient computational models of continuous models for numerical simulations, it is reasonable to study discrete time models governed by difference equations.

In 2003, Zhou and Zou [9] had studied a discrete logistic equation

$$x(n+1) = x(n)\exp\left\{r(n)\left[1 - \frac{x(n)}{K(n)}\right]\right\}. \tag{4}$$

When $r(n)$ and $K(n)$ are positive ω-periodic sequences, sufficient conditions are obtained for the existence of a positive and globally asymptotically stable ω-periodic solution of (4). In 2009, under the assumptions of almost periodicity of $r(n)$ and $K(n)$, Li and Chen [10] obtained the existence of a unique globally attractive almost periodic solution for (4).

Since the theory of delay differential equations was initiated by taking into account the fact that the behavior of physical systems not only depends on their present state, but also on their past history, time delays of one type or another have been incorporated into biological models to represent resource regeneration times, maturation periods, feeding times, reaction times, etc., by many researchers. We refer to the monographs of Cushing [11], Gopalsamy [12], Kuang [13] and Macdonald [14] for discussions of general delayed biological systems.

Sun and Li [15] studied the qualitative behavior of solutions of the discrete Logistic equation with several delays

$$x(n+1) = x(n)\exp\left\{\sum_{i=1}^{m} r_i\left[1 - \frac{x(n-k_i)}{K}\right]\right\}. \tag{5}$$

They obtained sufficient conditions for the global attractivity of all positive solutions about the positive equilibrium. Furthermore, they proved that such model is uniformly persistent and that all its eventually positive solutions are bounded. The oscillation about the positive equilibrium was also discussed.

Muroya [16] considered the following discrete model of a nonautonomous delay logistic equation

$$x(n+1) = x(n)\exp\left\{r(n) - \sum_{j=0}^{m} a_j(n)x(n-j)\right\}. \tag{6}$$

By using some kind of iterative method to (6), they established sufficient conditions that ensure the global attractivity for the solutions of (6).

On the existence of almost periodic sequence solutions for the discrete logistic model, some results are found in the literature. We refer to [10,17]. However, because of the introduction of the time delays, it is difficult to obtain the existence of almost periodic sequence solutions for discrete logistic model with delays (5)–(6) by the methods in [10,17]. To the best of our knowledge, up to now, there have been few papers concerning the existence of almost periodic sequence solutions for discrete delay logistic model (5)–(6). Feedback control is the basic mechanism by which systems, whether mechanical, electrical, or biological, maintain their equilibrium or homeostasis. In the higher life forms, the conditions under which life can continue are quite narrow. A change in body temperature of half a degree is generally a sign of illness. The homeostasis of the body is maintained through the use of feedback control [18]. A primary contribution of C.R. Darwin during the last century was the theory that feedback over long time periods is responsible for the evolution of species. In 1931 V. Volterra explained the balance between two populations of fish in a closed pond using the theory of feedback. Later, a series of mathematical models have been established to describe the dynamics of feedback control systems. Among these mathematical models, the logistic feedback control models play a fundamental and important role. In 1992, Gopalsamy [12] introduced a feedback control variable into the logistic models and discussed the asymptotic behavior of solutions in logistic models with feedback controls, in which the control variables satisfy a certain differential equation. We also refer to [19–27] for further study on equations with feedback control. The aim of this paper is, by applying the theory of difference inequality and constructing a suitable Lyapunov functional, to obtain the permanence and existence of a unique globally attractive positive almost periodic sequence solution of system (1).

Obviously, (4)–(6) are special cases of system (1).