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Robust $\mathcal{H}_\infty$ control for uncertain Lur'e systems with sector and slope restricted nonlinearities by PD state feedback[☆]

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ABSTRACT

In this paper, the design of a PD controller for robust $\mathcal{H}_\infty$ stabilization of uncertain Lur'e systems with sector and slope restricted nonlinearities is considered. A PD controller is utilized to not only guarantee stability of systems but also reduce the effect of external disturbance to a $\mathcal{H}_\infty$ norm constraint. Sector bounds and slope bounds are used on a Lyapunov–Krasovskii functional through convex representation of the nonlinear function so that a less conservative $\mathcal{H}_\infty$ stability criterion is obtained. Using Finsler's Lemma, the criterion is derived in terms of linear matrix inequalities (LMIs) that can be easily solved by various convex optimization techniques. Several examples show the effectiveness of the proposed methods.

1. Introduction

In recent years, Lur'e systems with a sector bound which was introduced by Lur'e in [1] have been extensively studied, in [2–7]. Since time delays are frequently encountered in many industrial and practical systems such as nuclear reactors, many researchers have focused on the stability of the time delay systems. For details, see the works [8–15] and references therein. In particular, the problem of stability for time delay Lur'e systems has been investigated [16–19]. In [16], Lee et al. studied the absolute stability for a time delay Lur'e system by using sector bound and slope bound information for the nonlinear function. In [17], Cao obtained a delay-dependent condition of stability of a Lur'e system by utilizing the descriptor system approach. On the other hand, one is often faced with model uncertainties and a lack of statistical information on the signals in real physical systems. This had led to an interest in min–max control, with the belief that $\mathcal{H}_\infty$ control is more robust and less sensitive to disturbance variances and model uncertainties. During the past few years, $\mathcal{H}_\infty$ control analysis and synthesis of control systems have received considerable attention, and a large number of excellent results such as [20–23] have been reported.

Proportional–integral–derivative (PID) control has a long history in control engineering. Its simplicity in architecture makes it much easier for the control engineer to understand and use than advanced controllers. But the full PID controller is not often used in the literature on stabilization for control systems. Hua and Guan [24] transformed chaotic systems with PI controllers into an augmented proportional control system. However, their methodology is not applicable to Lur'e systems

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with PD or PID controllers. In the area of PID controllers, it is well known that the derivative control is to enhance the stability. This implies that the derivative control is desirable for increasing stabilization of speed for control systems.

In this work, we deal with the problem of robust $\mathcal{H}_\infty$ control for uncertain Lur'e systems with sector and slope restricted nonlinearities by PD state feedback. The proposed controller design technique is based on a PD control and its gain matrices K_p, K_d are obtained by solving LMIs that are derived from a stability condition and the $\mathcal{H}_\infty$ norm bound of uncertain Lur'e systems. The nonlinear function is written as a convex representation of sector bounds and slope bounds, so the equality constraint is derived by using convex properties of the nonlinearities. A novel robust $\mathcal{H}_\infty$ criterion is obtained via LMIs by applying Finsler's Lemma [25]. Some examples show the effectiveness of the proposed method.

Notation: $\mathbb{R}^n$ denotes n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. $\mathbb{N}^+$ is the set of all positive integers. $A < 0$ means a real symmetric negative definitive matrix. I is the identity matrix with appropriate dimensions. diag$\{\cdots\}$ denotes the block diagonal matrix.

2. Preliminaries

Consider the following uncertain time delay Lur'e system with sector and slope restricted nonlinearities and a PD controller:

System :

$$\begin{aligned}&\dot{x}(t) = (A + \delta A(t))x(t) + (B + \delta B(t))x(t - \tau) + (F + \delta F(t))f(v(t)) + Cu(t) + Dw(t),\\ &v(t) = Hx(t),\end{aligned} \tag{1}$$

Controller :

$$u(t) = K_p x(t) + K_d \dot{x}(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^r$ is a control input, which will be appropriately designed such that the specific control objective is achieved, $K_p \in \mathbb{R}^{r\times n}$, $K_d \in \mathbb{R}^{r\times n}$ are the gain matrices for the PD controller, $w(t) \in \mathbb{R}^l$ is the external disturbance. $A \in \mathbb{R}^{n\times n}$, , $B \in \mathbb{R}^{n\times n}$, $F \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{n\times r}$, $D \in \mathbb{R}^{n\times l}$ and $H \in \mathbb{R}^{m\times n}$ are constant matrices. $v(t) \in \mathbb{R}^m$ is the output vector. The time delay τ is constant. $\delta A(t), \delta B(t), \delta F(t)$ are time-varying uncertainties with appropriate dimensions.

The uncertainties $\delta A(t), \delta B(t), \delta F(t)$ are assumed to be of the following form:

$$[\delta A(t) \quad \delta B(t) \quad \delta F(t)] = WG(t)[N_1 \quad N_2 \quad N_3], \tag{3}$$

where W, N_1, N_2 and N_3 are known real constant matrices with appropriate dimensions and $G(t)$ is an unknown real time-varying matrix satisfying

$$\|G(t)\| \le 1, \quad \forall t. \tag{4}$$

We assume that $f(v) \hat{=} [f_1(v_1(t)), \quad f_2(v_2(t)), \quad \ldots, \quad f_m(v_m(t))]^T$ is restricted by the sector bounds $[b_i, a_i]$ and the slope bounds $[\hat{b}_i, \hat{a}_i]$, i.e.

$$b_i \le \frac{f_i(v_i(t))}{v_i(t)} \le a_i, \tag{5}$$

$$\hat{b}_i \le \frac{\mathrm{d}(f_i(v_i(t)))}{\mathrm{d}(v_i(t))} \hat{=} f_i'(v_i(t)) \le \hat{a}_i, \quad i = 1, 2, \ldots m. \tag{6}$$

Remark 1. The constants a_i, b_i are allowed to be positive, negative or zero. Hence, the resulting condition is more general than the usual sector condition in [17].

The nonlinear function $f_i(\cdot)$ can be written as a convex combination of the sector bounds such as a_i, b_i:

$$f_i(v_i(t)) = (\Lambda_i^u(v_i) b_i + \Lambda_i^l(v_i) a_i) v_i(t), \tag{7}$$

where

$$\Lambda_i^l(v_i) = \frac{f_i(v_i(t)) - b_i v_i(t)}{(a_i - b_i) v_i(t)}, \qquad \Lambda_i^u(v_i) = \frac{a_i v_i(t) - f_i(v_i(t))}{(a_i - b_i) v_i(t)}.$$

Since $\Lambda_i^l(v_i(t)) + \Lambda_i^u(v_i(t)) = 1$, $\Lambda_i^u(v_i(t)) \ge 0$ and $\Lambda_i^l(v_i(t)) \ge 0$, the nonlinearity $f_i(\cdot)$ can be rewritten as

$$f_i(v(t)) = \Lambda_i(v_i(t)) v_i(t), \tag{8}$$

where $\Lambda_i(v_i(t))$ is an element of a convex hull $Co\{b_i, a_i\}$.

Similarly, the nonlinearity can also be expressed as a convex combination of the slope bounds such as $\hat{a}_i, \hat{b}_i$:

$$\dot{f}_i(v(t)) = \bar{\Lambda}_i(v_i(t)) \dot{v}_i(t), \tag{9}$$

where $\bar{\Lambda}_i(v_i(t)) \in Co\{\hat{b}_i, \hat{a}_i\}$.