Nonlinear Analysis: Real World Applications 12 (2011) 731–740

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

A regularization smoothing method for second-order cone complementarity problem ☆

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ARTICLE INFO

Article history:
Received 26 February 2009
Accepted 3 August 2010

Keywords:
Second-order cone complementarity problem
Regularization method
Smoothing Newton method
Global convergence
Local superlinear convergence

ABSTRACT

In this paper, the second-order cone complementarity problem is studied. Based on the Fischer–Burmeister function with a perturbed parameter, which is also called smoothing parameter, a regularization smoothing Newton method is presented for solving the sequence of regularized problems of the second-order cone complementarity problem. Under proper conditions, the global convergence and local superlinear convergence of the proposed algorithm are obtained. Moreover, the local superlinear convergence is established without strict complementarity conditions. Preliminary numerical results suggest the effectiveness of the algorithm.

1. Introduction

The second-order cone complementarity problem (SOCCP in short) is to find a vector $z \in R^n$ such that

$$\langle f(z), z\rangle = 0 \quad \text{and} \quad f(z) \in K, \quad z \in K, \tag{1}$$

where $\langle \cdot, \cdot\rangle$ represents the Euclidean inner product, $f : R^n \to R^n$ is a continuously differentiable mapping, and K is the Cartesian product of second-order cones, that is $K = K^{n_1} \times K^{n_2} \times \cdots \times K^{n_m}$ with $n_1 + n_2 + \cdots + n_m = n$ and $n_1, n_2, \ldots, n_m \geq 1$. The n_i-dimensional second-order cone K^{n_i} is defined by

$$K^{n_i} := \{(z_1, z_2^T)^T \in R \times R^{n_i - 1} | z_1 \geq \|z_2\|\},$$

where $\|\cdot\|$ denotes the Euclidean norm and K^1 denotes the set of nonnegative reals R_+ (the nonnegative orthant in R). When $n_1 = n_2 = \cdots = n_m = 1$, it can be seen that SOCCP is equivalent to the nonlinear complementarity problem (NCP). Additionally, the Karush–Kuhn–Tucker(KKT) conditions for any second-order cone programming with continuously differentiable functions can also be written in the form of SOCCP; see Ref. [1].

Recently, the second-order cone complementarity problem has drawn a lot of attention partially due to its wide applications [1–4]. Analogous to the nonlinear complementarity problem and the semidefinite complementarity problem, the second-order cone complementarity problem can be employed for a reformulation of (1) as an unconstrained smooth minimization problem or a system of nonlinear equations to solve. Some methods have been developed to treat it [5–7], but most of their algorithms depend on the assumptions of monotone or strict complementarity. Moreover, there is little work for solving the singular second-order cone complementarity problem, in which the derivative of the mapping may be

☆ This work is supported by the National Science Foundation of China (Grant No. 60674108) and Basic Science Research Fund in Xidian University.

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1468-1218/$ – see front matter © 2010 Published by Elsevier Ltd
doi:10.1016/j.nonrwa.2010.08.001

seriously ill-conditioned. These motivated us to study this class of problems and obtain a method to try to circumvent one or several difficulties in their algorithms.

As we know, there are two classes of methods to handle the singular nonlinear complementarity problems: regularization methods [8,9] and proximal point methods [10,11], see the report [12] and the references therein for details. In this paper, we discuss the class of regularization methods for the second-order cone complementarity problem. This class of methods can deal with the singularity problem by considering a sequence of perturbed problems which possibly have better conditions. The simplest regularization technique is the so-called Tikhonov regularization, which involves solving a sequence of complementarity problems, i.e.,

$$\langle f_\mu(z), z\rangle = 0 \quad \text{and} \quad f_\mu(z) \in K, \quad z \in K,$$

where μ is a positive parameter tending to zero and $f_\mu(z) = f(z) + \mu z$. Based on this regularization method, Sun proposed a regularization Newton method for solving the nonlinear complementarity problems with a P_0 function in [9]. It was shown that the proposed algorithm [9] does not require the strict complementarity condition in the local superlinear (quadratic) convergence. Lately, Chen and Ma gave a new regularization smoothing method for the nonlinear complementarity problem in [13], i.e.,

$$f_\mu(z) = f(z) + \frac{1}{2}\mu e^{\sin z}.$$

This new regularization method can smooth the complementarity function, but the Tikhonov-regularization method cannot. Considering the virtues of the regularization technique of Chen et al. and the algorithm of Sun, we shall combine and extend them to solve the second-order cone complementarity problem.

In this paper, by using a new regularization method, we reformulate the SOCCP into a system of nonlinear equations based on the Fischer–Burmeister function, and present a regularization smoothing Newton method for solving the sequence of problems approximately. The proposed algorithm only solves a linear system of equations and performs only one line search at each iteration. We prove the global convergence and local superlinear convergence of the algorithm. Furthermore, in the absence of a strict complementarity condition, we establish the local superlinear convergence of the algorithm under the assumption of nonsingularity. To evaluate the efficiency of the algorithm, we conduct some numerical experiments.

This paper is organized as follows. In the next section, some preliminaries with second-order cones are introduced first, then a complementarity function is studied and some definitions are included. In Section 3, a regularization smoothing Newton algorithm is presented. In Section 4, the global convergence and local convergence of the algorithm are discussed. Numerical results are reported in Section 5. Some conclusions are given in Section 6.

Throughout this paper, all vectors are column vectors, ^{T} denotes transpose, I represents an identity matrix of suitable dimension, and $\|\cdot\|$ denotes the Euclidean norm defined by $\|x\| := \sqrt{x^Tx}$ for a vector x. R_{++} means the positive orthant of R. For any differentiable function $f : R^n \to R^n$, $\nabla f(x)$ denotes the gradient of f at x. Let int$K$ denote the interior of K. $x \succeq y$ or $x \succ y$ means that $x - y \in K$ or $x - y \in \text{int}K$, respectively. For simplicity, we use $x = (x_1, x_2) \in R \times R^{n-1}$ for the column vector $x = (x_1, x_2^T)^T$.

2. Preliminaries

2.1. Jordan algebra associated with SOC

In this subsection, we shall give the basic facts concerning Euclidean–Jordan algebra [1,14], which provides a useful methodology of dealing with second-order cone (SOC for short).

A Euclidean–Jordan algebra is a triple $(V, \langle\cdot,\cdot\rangle, \circ)$ (V for short), where $(V, \langle\cdot,\cdot\rangle)$ is a finite-dimensional inner product space over R and $(x, y) \mapsto x \circ y : V \times V \to V$ is a bilinear mapping which satisfies the following conditions:

(a) $x \circ y = y \circ x$, for any $x, y \in V$.
(b) $x \circ (x^2 \circ y) = x^2 \circ (x \circ y)$ for all $x, y \in V$ where $x^2 = x \circ x$.
(c) $\langle (x \circ y, z)\rangle = \langle (x, y \circ z)\rangle$ for all $x, y, z \in V$.

In this paper, we consider R^n with the Euclidean–Jordan algebra $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$. In this algebra, the SOC K is the cone of squares, i.e., $K = \{x^2 : x \in (R^n, \langle\cdot,\cdot\rangle, \circ)\}$. For any $x = (x_1, x_2), y = (y_1, y_2) \in R^n$, their Jordan product associated with K is defined by

$$x \circ y := (x^Ty, x_1y_2 + y_1x_2).$$

Some of the prominent relations involving the binary operation $\circ$ are as follows,

(a) the vector $\bar{e} = (1, 0, \ldots, 0)^T \in R^n$ is the unique identity element: $x \circ \bar{e} = x$.
(b) Write x^2 to mean $x \circ x$ and $x + y$ for the usual componentwise addition of vectors.
(c) $x^2 \in K$, for all $x \in R^n$.
(d) If $x \in K$, there exists a unique vector in K, denoted by $x^{1/2}$, such that $(x^{1/2})^2 = x^{1/2} \circ x^{1/2} = x$.