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# Robust stabilization of delayed nonholonomic systems with strong nonlinear drifts

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ABSTRACT

This paper presents a control strategy for stabilization of nonholonomic control systems with strongly nonlinear uncertainties and time delay. By applying a novel Lyapunov functional, discontinuous transformation and dynamic feedback approach, robust nonlinear state feedback switching controllers are designed, which can guarantee the stabilization of closed loop systems. The proposed method proves to be effective by a simulation example.



## 1. Introduction

In the last few decades, there has been a rapidly growing research interest in nonholonomic system, which is a particular class of nonlinear systems and can be found frequently in the real world, such as mobile robots, car-like vehicle, under-actuated satellites, the knife-edge and so on. As pointed out in [1,2], such a class of nonlinear systems cannot be asymptotically stabilized at the origin by only using continuous state feedback control signal. Therefore, control of nonholonomic systems is extremely challenging. To date, several controller designs have been proposed to achieve the asymptotic stabilization or exponential regulation for nonholonomic control systems [3–16] such as smooth time-varying feedback control strategies in [4–7], and discontinuous feedback techniques in [8–10].

As shown in [4], many nonlinear mechanical systems with nonholonomic constraints on velocities can be transformed, either locally or globally, to driftless nonholonomic systems in the so-called chained form. There have been several research works [4–7,17,18] for such a class of nonholonomic systems. Recently, much attention [12–16] has been paid to the nonholonomic systems with drift uncertain nonlinearity for considering the possible modeling errors and external disturbance.

However, most modelings considered are in ideal cases. From a practical point of view, when modeling a mechanical system, time delay should be taken into account. On the other hand, time delay in systems is always unknown, which may result in large obstacles for control problems. How to overcome the difficulties from the existence of the time delay, will be a challenging problem. To the best of the authors' knowledge, there is no result for the nonholonomic systems with time delay.

In this paper, we introduce a new class of chained nonholonomic systems with drift uncertain nonlinearities and time delay, and then study the problem of robust state feedback stabilization for the concerned nonholonomic systems. Since the nonholonomic system considered in this note contains the time delay, it cannot be handled by general existing methods. The constructive design method proposed in this note is based on a combined application of the novel discontinuous

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transformation technique, the dynamic feedback approach and the new Lyapunov functional. The switching control strategy for the first subsystem is employed to achieve the robust asymptotic stabilization.

**Notation.** Throughout this paper, $\|\cdot\|$ denotes the Euclidean norm. A real symmetric matrix $X > 0 (\geq 0)$ denotes $X$ being a positive definite (positive semi-definite) matrix. $\lambda_{\max}(X)$ and $\lambda_{\min}(X)$ represent for the maximum and minimum eigenvalues of the matrix $X$, individually. $A = (a_{ij})_{m\times n}$ is a $m \times n$ matrix, then denote $|A| = (|a_{ij}|)_{m\times n}$. $I$ is used to denote an identity matrix with proper dimension.

## 2. Problem formulation and preliminaries

Consider the following nonholonomic system with strong nonlinear drifts and time delay

$$\begin{aligned}
\dot{x}_0(t) &= u_0(t) + \lambda_0 x_0(t),\\
\dot{x}_1(t) &= u_0(t)x_2(t) + \phi_1^d(t, x_0(t), x(t), x(t-\tau)),\\
&\vdots\\
\dot{x}_i(t) &= u_0(t)x_{i+1}(t) + \phi_i^d(t, x_0(t), x(t), x(t-\tau)),\\
&\vdots\\
\dot{x}_n(t) &= u_1(t) + \phi_n^d(t, x_0(t), x(t), x(t-\tau))
\end{aligned} \tag{1}$$

where $[x_0(t), x(t)]^T = [x_0(t), x_1(t), \ldots, x_{n-1}(t)]^T \in R^n$ and $u(t) = [u_0(t), u_1(t)]^T \in R^2$ are system state and control input, respectively; the functions $\phi_i^d(\cdot)$'s are regarded as the input and state-driven uncertainty delayed functions; $\tau$ is a unknown time delay satisfying $\tau \leq \bar{\tau}$, and $\bar{\tau}$ is a known constant.

Suppose that the system (1) satisfies the following assumption, which will be the base of the coming control design and performance analysis.

**Assumption 1.** For every $1 \leq i \leq n$, there exit nonnegative constants $\delta_{ijl}$, $\alpha_{ijl}$, $\beta_{ijl}$, $C_{il}$ and positive integers $N_i$ such that the following inequality holds

$$|\phi_i^d(t, x(t), x(t-\tau))| \leq \sum_{l=1}^{N_i} C_{il} \prod_{j=1}^{n} \left(|x_0(t)|^{\delta_{ijl}} |x_j(t)|^{\alpha_{ijl}} |x_j(t-\tau)|^{\beta_{ijl}}\right). \tag{2}$$

It is further assumed that $\delta_{ijl}$, $\alpha_{ijl}$ and $\beta_{ijl}$ satisfy $\sum_{j=1}^{n}[\delta_{ijl} + j(\alpha_{ijl} + \beta_{ijl})] \leq i$, $\sum_{j=1}^{n}(\alpha_{ijl} + \beta_{ijl}) \geq 1$, and $\sum_{j=1}^{n} \beta_{ijl} \leq 1$ for $\forall l$.

**Remark 1.** From the system (1), it can be observed that the delay part $x_i(t-\tau)$, as well as the part $x_i(t)$, exists not only in $x_j$-subsystems ($j \geq i$), it is also in $x_j$-subsystems ($j \leq i$). Then, the system considered in this paper is more general and has better applicability.

**Remark 2.** In this paper, Assumption 1 is imposed on the uncertain delayed functions of system (1). From (2), we can see that the backstepping recursive approach widely used in [12,16] cannot be effective for system (1). In this note we will resort to the novel dynamic feedback approach to construct the controller for the system (1).

**Lemma 1.** *There exist constants $a_i$ ($i = 1, 2, \ldots, n$), $\alpha > 0$ and matrix $Q > 0$ such that*

$$QA + A^TQ \leq -I, \qquad QB + B^TQ \geq \alpha I, \tag{3}$$

*where*

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix},$$

$$B = \mathrm{diag}\{1, 2, \ldots, n\}.$$

**Remark 3.** Lemma 1 can be derived from the Lemma 1 in [19] easily, then the proof is omitted.

## 3. Controller design

In this section, we use two separate stages to globally asymptotically stabilize the system (1). Firstly, the control $u_0(t)$ should be designed for $x_0$-subsystem; in the second stage, we design $u_1(t)$ to guarantee all states of the rest in system (1) convergence to zero.