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Traveling waves in delayed lattice dynamical systems with competition interactions☆

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ABSTRACT

This paper deals with the existence of traveling wave solutions of a class of delayed system of lattice differential equations, which formulates the invasion process when two competitive species are invaders. Employing the comparison principle of competitive systems, a new cross-iteration scheme is given to establish the existence of traveling wave solutions. More precisely, by the cross-iteration, the existence of traveling wave solutions is reduced to the existence of an admissible pair of upper and lower solutions. To illustrate our main results, we prove the existence of traveling wave solutions in two delayed two-species competition systems with spatial discretization. Our results imply that the delay appeared in the interspecific competition terms do not affect the existence of traveling wave solutions.

1. Introduction

Lattice dynamical systems are infinite systems of ordinary differential equations (so-called lattice ODE) or of difference equations (so-called coupled map lattices), indexed by points in a lattice, such as the D-dimensional integer lattice $\mathbb{Z}^D$ which incorporate some aspect of the spatial structure of the lattice. Such systems arise, on the one hand, from practical backgrounds, such as modeling the population growth over patchy environments [1–3] and the phase transitions [4,5]. On the other hand, they also arise as the spatial discretization of partial differential equations; we refer to [6–10].

In the past decades, more and more evidence indicates that the traveling wave solutions play an important role in the study of lattice dynamical systems. More precisely, the traveling wave solutions can determine the long term behavior of the corresponding initial value problems of lattice dynamical systems, which partly arise from the stability of traveling wave solutions; e.g., we can refer to [11–15]. At the same time, the traveling wave solutions in lattice differential equations may describe many important phenomena in physical systems, population dynamics and other fields; see [4,16,17] and the references cited therein.

For the *single* lattice differential equation, a typical example is

$$\frac{\mathrm{d}u_n(t)}{\mathrm{d}t} = D\left(u_{n+1}(t) - 2u_n(t) + u_{n-1}(t)\right) + f(u_n(t)), \quad n \in \mathbb{Z}, t > 0, \tag{1.1}$$

which was initially used in Bell and Cosner [1] to model myelinated axons in nerve systems. They studied the long time behavior of solutions to (1.1) for some nonlinear function f. The traveling wave solutions were analytically discussed in [18] and numerically computed in [19]. Keener [2] analyzed propagation and its failure for (1.1). In particular, when the

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nonlinear term in (1.1) is of bistable type, the study on traveling wave fronts of such lattice differential equations is extensive and intensive, and has resulted in many interesting and significant results, some of which have revealed some essential differences between a discrete model and its continuous version—parabolic equations (see [20]). For details, see, for example, [1,2,4,5,8,21–25], and the references therein. When the nonlinear term is of monostable type, Zinner et al. [26] addressed the existence and minimal speed of traveling wave front for discrete Fisher equation. Recently, Chen and Guo [12,27] discussed a more general case of equation

$$\frac{\mathrm{d}u_n(t)}{\mathrm{d}t} = D\left(g(u_{n+1}(t)) - 2g(u_n(t)) + g(u_{n-1}(t))\right) + f(u_n(t)), \quad n \in \mathbb{Z}, t > 0, \tag{1.2}$$

where $g(u)$ is increasing and $f(u)$ is monostable, and the authors established the existence, uniqueness and stability as well as minimal wave speed for (1.2).

It is well known that, in modeling population growth and transition of signals in the nerve systems, temporal delays seem to be inevitable, accounting for the maturation time of the species under consideration and the time needed for the signals to travel along axons and to cross synapses. Based on such a consideration, Wu and Zou [28] first studied the existence of traveling wave solutions of delayed lattice differential equations. Liang and Zhao [29] further considered the spatial–temporal pattern of these models by monotone dynamical systems. For more results, see [3,7,13,14,30–35].

For *systems* of lattice differential equations, a simple example is

$$\begin{cases} \dfrac{\mathrm{d}u_n(t)}{\mathrm{d}t} = d_1\left(u_{n+1}(t) - 2u_n(t) + u_{n-1}(t)\right) + u_n(t)(r_1 - b_1 v_n(t)), \\ \dfrac{\mathrm{d}v_n(t)}{\mathrm{d}t} = d_2\left(v_{n+1}(t) - 2v_n(t) + v_{n-1}(t)\right) + v_n(t)(-r_2 + b_2 u_n(t)), \end{cases}$$

which was proposed by Renshaw [36] to model morphogenesis growth, and is also called the Turing model. In 1995, Anderson and Sleeman [37] investigated a spatially discretized Fitzhugh–Nagumo system with monotone reaction terms, and studied the existence and propagation failure of traveling wave fronts; also see Nekorkin et al. [38]. Recently, Huang and Lu [39] and Huang et al. [40] considered the following delayed lattice systems

$$\begin{cases} \dfrac{\mathrm{d}u_n(t)}{\mathrm{d}t} = \displaystyle\sum_{j=1}^{m} a_j[g(u_{n+j}(t)) - 2g(u_n(t)) + g(u_{n-j}(t))] + f_1(u_{nt}, v_{nt}), \\ \dfrac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \displaystyle\sum_{j=1}^{m} b_j[g(v_{n+j}(t)) - 2g(v_n(t)) + g(v_{n-j}(t))] + f_2(u_{nt}, v_{nt}), \end{cases}$$

where $n \in \mathbb{Z}$, $t > 0$, u_n and v_n are continuous functions, g is a continuous function, f_1 and f_2 are continuous functions defined on functional space $C([-\tau, 0], \mathbb{R}^2)$ and valued in $\mathbb{R}$, $u_{nt} \in C([-\tau, 0], \mathbb{R})$ defined by $u_n(t+s)$ is a continuous function for $s \in [-\tau, 0]$, so for v_{nt}. By using the idea of Huang et al. [31] and Wu and Zou [28] for delayed lattice differential equations, Huang and Lu [39], Huang et al. [40] and Lin et al. [41] established the existence of traveling wave solutions connecting trivial equilibrium $(0, 0)$ with nontrivial one (k_1, k_2), if the reaction terms satisfy the so-called (exponential) quasimonotone condition or the partial (exponential) quasimonotone condition. For related results on reaction–diffusion equations with delays, we refer to [42–53] and references cited therein.

However, it is quite common that the reaction terms in some models may not satisfy the monotone conditions mentioned above, such as the following two systems of lattice ODEs

$$\begin{cases} \dfrac{\mathrm{d}u_n(t)}{\mathrm{d}t} = d_1[u_{n+1}(t) - 2u_n(t) + u_{n-1}(t)] + r_1 u_n(t)\left[1 - a_1 u_n(t) - b_1 v_n(t-\tau_1)\right], \\ \dfrac{\mathrm{d}v_n(t)}{\mathrm{d}t} = d_2[v_{n+1}(t) - 2v_n(t) + v_{n-1}(t)] + r_2 v_n(t)\left[1 - b_2 u_n(t-\tau_2) - a_2 v_n(t)\right], \end{cases} \tag{1.3}$$

and

$$\begin{cases} \dfrac{\mathrm{d}u_n(t)}{\mathrm{d}t} = d_1[u_{n+1}(t) - 2u_n(t) + u_{n-1}(t)] + r_1 u_n(t)\left[1 - a_1 u_n(t-\tau_1) - b_1 v_n(t-\tau_2)\right], \\ \dfrac{\mathrm{d}v_n(t)}{\mathrm{d}t} = d_2[v_{n+1}(t) - 2v_n(t) + v_{n-1}(t)] + r_2 v_n(t)\left[1 - b_2 u_n(t-\tau_3) - a_2 v_n(t-\tau_4)\right], \end{cases} \tag{1.4}$$

which can be regarded as spatially discrete versions of the diffusion competition systems with delays considered in [54]. It is evident that (1.3) and (1.4) do not satisfy the monotone conditions in [28,29,31,39–41] when we are interested in the traveling wave solutions connecting $(0, 0)$ with positive equilibrium (if it exists). Furthermore, (1.3) and (1.4) can satisfy the monotone condition in [29,31,41] by change of variables, but the new interested equilibria are not ordered such that the known results fail. Due to the ecology sense of trivial and coexistence equilibria of (1.3) and (1.4), such a traveling wave solution is very important in modeling the population invasions when two species are competitive invaders. Therefore, it is worthwhile to further explore this topic for lattice differential systems including (1.3) and (1.4), and this constitutes the purpose of this paper. For the related topics in reaction–diffusion systems, we refer to [46,54–57].