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A new model transformation method and its application to extending a class of stability criteria of neutral type systems

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ABSTRACT

This paper proposes a generalized equivalent model transformation method, which can include methods proposed by Fridman et al. and Bellen et al., for the stability analysis of a class of neutral type systems. By using the proposed model transformation method, a class of existing stability criteria derived by the Lyapunov functional approach can be extended to less conservative ones in terms of nonlinear matrix inequalities. Furthermore, procedures to solve these nonlinear matrix inequalities are also proposed. Illustrative examples are presented to demonstrate the effectiveness of the proposed model transformation method.

1. Introduction

Recently, many model transformation methods have been proposed for the stability analysis of neutral type systems [1–3]. However, some of the transformed systems are not equivalent to the original systems. In order to overcome the problem, Fridman et al. proposed an equivalent augmented model as a "descriptor form" representation of the system [2,3]. Another similar equivalent model transformation method was proposed by Bellen et al. [4].

Both models proposed by Fridman et al. and Bellen et al. are in fact special cases of the 2D model [5–7]. Based on this fact, we introduce a slack matrix into the equivalent augmented model proposed by Fridman et al. to form a generalized 2D model. Through choosing specific slack matrices, the proposed generalized model can recover equivalent models proposed by Fridman et al. and Bellen et al. The new model transformation method can be used to extend many existing stability criteria. However, in order to demonstrate the effectiveness more explicitly, we only focus on extending a class of existing stability criteria. By using this new model transformation method and the Lyapunov functional approach, the class of existing stability criteria can be extended to less conservative criteria in terms of nonlinear matrix inequalities. In view of this, procedures to solve these nonlinear matrix inequalities are also proposed. The effectiveness of the proposed model transformation method is demonstrated through illustrative examples. The main contributions of this paper are: (1) a new model transformation method is proposed; (2) based on this new transformation method, this paper extends a class of existing stability criteria rather than just designing new Lyapunov functionals, and it is proven that the extended criteria can reduce the conservatism of the original criteria; (3) procedures to solve a class of nonlinear matrix inequalities are proposed.

The notation used in this paper is as follows. $\mathbb{R}^n$ is the Euclidean space of dimension n. X^T is the transpose of matrix X. $|\cdot|$ denotes the absolute value of a scalar and $\|\cdot\|$ denotes the Euclidean norm or the matrix norm induced by the Euclidean norm. $\lambda_{\max}(X)$ denotes the maximal eigenvalue of matrix X. $X > 0$ $(X < 0)$ represents that matrix X is a positive definite (negative definite) matrix. I_n is an identity matrix of a specified dimension n. $0_{n\times m} \in \mathbb{R}^{n\times m}$ denotes a zero matrix.

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2. A new model transformation method

Given the following neutral type system:

$$0 = F\left(A_0, A_1, x, \dot{x}, t\right) \tag{1}$$

with the initial condition:

$$x(t) = \phi(t), \quad \forall t \in [-\tau, 0]$$

where

$$F\left(X_0, X_1, x, v, t\right) \triangleq v(t) - Dv(t-\tau) - X_0 x(t) - X_1 x(t-\tau) - f(x,t)$$
$$x(t), v(t) \in \mathbb{R}^{n\times 1}, X_0, X_1 \in \mathbb{R}^{n\times n}, f(x,t) \in \mathbb{R}^{n\times 1}, \tau \in \mathbb{R}.$$

In (1), $X_0 = A_0, X_1 = A_1, v(t) = \dot{x}(t)$. $x(t)$ is the state vector, τ is the time delay, $f(x,t)$ is a vector function which does not contain derivative terms of the state vector, $\phi(t)$ is a continuously differentiable smooth vector valued function representing the initial condition function. If $D = 0$, then system (1) degenerates into a retarded type system. Without loss of generality, the purpose of this paper is to propose a new model transformation method to derive stability criteria for neutral type system (1).

Define

$$y(t) \triangleq \dot{x}(t) - Sx(t) \tag{2}$$

where $S \in \mathbb{R}^{n\times n}$ is the slack matrix which needs to be designed to obtain less conservative stability criteria. This will be discussed later.

Since the vector function $f(x,t)$ does not contain derivative terms of the state vector, substituting $\dot{x}(t) = Sx(t) + y(t)$ into (1) yields:

$$0 = [Sx(t) + y(t)] - D[Sx(t-\tau) + y(t-\tau)] - A_0 x(t) - A_1 x(t-\tau) - f(x,t).$$

Consequently, system (1) is transformed into the following equivalent form:

$$\begin{cases} \dot{x}(t) = Sx(t) + y(t) \\ 0 = F\left(A_0 - S, A_1 + DS, x, y, t\right) \end{cases} \tag{3}$$

where $y(t)$ can be treated as the 'fast variable' as mentioned in [2].

Since the transformed system (3) is equivalent to the original system, we will focus on the stability analysis of the transformed system in the following sections. When choosing $S = 0$ or $S = A_0$, system (3) can recover the equivalent models proposed in [2] or [4], respectively. Furthermore, by designing the appropriate slack matrix S, the conservatism of the criteria derived by the model transformation methods proposed in [2,4] can be effectively reduced.

3. A method to extend a class of stability criteria

The proposed model transformation method can help to design new Lyapunov functionals and then obtain new stability criteria as proposed in [2]. However, in order to demonstrate the effectiveness more explicitly, we only focus on extending a class of existing stability criteria in this section. First, by applying the proposed model transformation method, a simple application on extending an existing stability criterion is given. Following the idea of this application, a generalized method in terms of a theorem is derived to extend a class of existing stability criteria. Finally, a stability criterion proposed in [8] is extended to a less conservative one by using the generalized method.

3.1. A simple application

For simplicity, consider neutral type system (1) with $f(x,t) \equiv 0$, i.e.

$$0 = F_1\left(A_0, A_1, x, \dot{x}, t\right) \tag{4}$$

where $F_1\left(X_0, X_1, x, v, t\right) \triangleq v(t) - Dv(t-\tau) - X_0 x(t) - X_1 x(t-\tau)$.

The following criterion proposed in [2] is used to determine the delay-independent stability of neutral type system (4).

Fact 1 ([2]). *If there exist* $0 < P_1 = P_1^T, P_2, P_3 \in \mathbb{R}^{n\times n}$ *and* $0 < Q_1 = Q_1^T, 0 < Q_2 = Q_2^T \in \mathbb{R}^{n\times n}$ *such that:*

$$\Omega_1\left(A_0, A_1, \mathcal{P}_1\right) < 0 \tag{5}$$