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Hopf bifurcations in a predator–prey system with a discrete delay and a distributed delay

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ABSTRACT

A delayed Lotka–Volterra two-species predator–prey system with discrete hunting delay and distributed maturation delay for the predator population described by an integral with a strong delay kernel is considered. By linearizing the system at the positive equilibrium and analyzing the associated characteristic equation, the asymptotic stability of the positive equilibrium is investigated and Hopf bifurcations are demonstrated. It is found that under suitable conditions on the parameters the positive equilibrium is asymptotically stable when the hunting delay is less than a certain critical value and unstable when the hunting delay is greater than this critical value. Meanwhile, according to the Hopf bifurcation theorem for functional differential equations (FDEs), we find that the system can also undergo a Hopf bifurcation of nonconstant periodic solution at the positive equilibrium when the hunting delay crosses through a sequence of critical values. In particular, by applying the normal form theory and the center manifold reduction for FDEs, an explicit algorithm determining the direction of Hopf bifurcations and the stability of bifurcating periodic solutions occurring through Hopf bifurcations is given. Finally, to verify our theoretical predictions, some numerical simulations are also included at the end of this paper.
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1. Introduction

In recent years, a large number of population models, especially the Lotka–Volterra predator–prey models modeled by ordinary differential equations (ODEs), have been proposed and studied extensively since the pioneering theoretical works by Lotka [1] and Volterra [2]. To reflect that the dynamical behavior of the models depends on the past history of the system, it is often necessary to incorporate time delays into the models. Therefore, a more realistic predator–prey model should be described by delayed differential equations. Let $x(t)$ and $y(t)$ denote the population density of prey and predator at time t, respectively, and suppose that the predator population at every age stage has the ability to predate and that the prey population captured by the predator population in the past is all contributing to the predator population at time t; then in this case the growth dynamics of the two species can be described by the following delayed Lotka–Volterra two-species predator–prey system with distributed delays:

$$\begin{cases} \dot{x} = x(t)\left[r_1 - a_{11}x(t) - a_{12}\int_{-\infty}^{t} F(t-s)y(s)\right]\mathrm{d}s, \\ \dot{y} = y(t)\left[-r_2 + a_{21}\int_{-\infty}^{t} G(t-s)x(s)\mathrm{d}s - a_{22}y(t)\right], \end{cases} \tag{1.1}$$

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where the constants $r_1 > 0$ and $r_2 > 0$ denote the intrinsic growth rate of the prey species and the death rate of the predator species, respectively; a_{ij} $(i, j = 1, 2)$ are all positive constants, and $\frac{r_1}{a_{11}}$, a_{12}, a_{21} and a_{22} represent the carrying capacity of the prey population, the predation coefficient of the predator, the transformation rate of prey captured by the predator to the predator population and the overcrowding rate of the predator population itself, respectively; the bounded non-negative functions $F(s)$ and $G(s)$ are called the delay kernel, and they satisfy the following normalized conditions:

$$\int_0^\infty F(s)\mathrm{d}s = 1, \qquad \int_0^\infty G(s)\mathrm{d}s = 1. \tag{1.2}$$

Systems such as (1.1) with various delay kernels and delayed intraspecific competitions have been investigated extensively by many researchers; see [3–11]. For example, when system (1.1) has no effects of delays, that is, $F(s) = G(s) = \delta(s)$, where δ denotes Dirac delta function, Chen [3] and Zhang and Feng [11] showed that the existence of a positive equilibrium of (1.1) implies its global asymptotic stability. When $F(s) = \delta(s - \tau)(\tau \geq 0)$ and $G(s) = \delta(s - \eta)(\eta \geq 0)$, namely, system (1.1) has two different discrete delays, He [12] and Lu and Wang [13] investigated the stability of the positive equilibrium of the system, and they found that the positive equilibrium is globally asymptotically stable for any values of delays τ and η when the coefficients of the system satisfy the condition $a_{11}a_{22} - a_{12}a_{21} > 0$. In addition, under the condition that $\eta > 0$, by considering η as the bifurcation parameter and using the linearization method, Faria [4] investigated the stability of the positive equilibrium of system (1.1) and the Hopf bifurcation of nonconstant periodic solutions near the positive equilibrium, and the normal form of Hopf bifurcations was also given by using the normal form theory and the center manifold theorem developed by Faria and Magalhães [14]. For the same model, Ruan [5] and Yan and Chu [8] also investigated the stability of the positive equilibrium of system (1.1) and the Hopf bifurcation of nonconstant periodic solutions by regarding the sum of two delays τ and η as the bifurcation parameter. Furthermore, for the study of system (1.1) with delayed intraspecific competitions, one can refer to [6,9,10].

Assume that $F(s) = \delta(s - \tau)(\tau \geq 0)$; then system (1.1) is reduced to the following Lotka–Volterra two-species predator–prey system with a discrete delay and a distributed delay:

$$\begin{cases} \dot{x} = x(t)[r_1 - a_{11}x(t) - a_{12}y(t - \tau)], \\ \dot{y} = y(t)\left[-r_2 + a_{21}\displaystyle\int_{-\infty}^{t} G(t - s)x(s)\mathrm{d}s - a_{22}y(t)\right], \end{cases} \tag{1.3}$$

where the nonnegative constant τ can be interpreted as the hunting delay of the predator population. The delay kernel function $G(s)$ may take the so-called "weak" generic kernel function $G(s) = \alpha \mathrm{e}^{-\alpha s}$ $(\alpha > 0)$ and "strong" generic kernel function $G(s) = \alpha^2 s\mathrm{e}^{-\alpha s}$ $(\alpha > 0)$, where the "weak" generic kernel implies that the importance of events in the past simply decreases exponentially the further one looks into the past while the "strong" generic kernel implies that a particular time in the past is more important than any other [15]. In the case that $G(s)$ takes the "weak" generic kernel function $G(s) = \alpha \mathrm{e}^{-\alpha s}(\alpha > 0)$, Song and Yuan [7] investigated the stability of the positive equilibrium of system (1.3) and Hopf bifurcations of nonconstant periodic solutions by using the linearization method and regarding the discrete hunting delay τ as the bifurcation parameter. It is shown that the positive equilibrium of system (1.3) is asymptotically stable when the discrete hunting delay τ is less than a certain critical value and unstable when τ is greater than this critical value. In addition, by using the normal form theory and the center manifold reduction for FDEs, Song and Yuan [7] also studied the direction of the Hopf bifurcations and the stability of bifurcated periodic solutions occurring through Hopf bifurcations.

However, when the delay kernel $G(s)$ takes the "strong" generic kernel function $G(s) = \alpha^2 s\mathrm{e}^{-\alpha s}$ $(\alpha > 0)$, it is an unknown problem how the discrete hunting delay τ and the delay kernel $G(s) = \alpha^2 s\mathrm{e}^{-\alpha s}$ $(\alpha > 0)$ (i.e., the parameter α) affect the dynamics of system (1.3). Therefore, in this paper, we study mainly the effects of the hunting delay τ and the parameter α in the "strong" delay kernel on the dynamical behaviors of system (1.3). By means of the change of variables, we first transform system (1.3) with the "strong" delay kernel into a four-dimensional system of delayed differential equations with a single delay. Then, by linearizing the resulting four-dimensional system at the positive equilibrium and analyzing the associated characteristic equation, the asymptotic stability of the positive equilibrium is investigated and Hopf bifurcations are demonstrated. In particular, by applying the normal form theory and the center manifold reduction for FDEs due to Hassard, Kazarinoff and Wan [16], an explicit algorithm determining the direction of the Hopf bifurcations and the stability of bifurcated periodic solutions occurring through Hopf bifurcations is given.

This paper is organized as follows. In Section 2, by linearizing the resulting four-dimensional system at the positive equilibrium and analyzing the associated characteristic equation, the asymptotic stability of the positive equilibrium and the existence of Hopf bifurcations are investigated. In Section 3, to determine the direction of the Hopf bifurcations and the stability of bifurcated periodic solutions occurring through Hopf bifurcations, an explicit algorithm is given by applying the normal form theory and the center manifold reduction for FDEs developed by Hassard, Kazarinoff and Wan [16]. To verify our theoretical predictions, some numerical simulations are also included in Section 4.

2. Stability of equilibria and existence of Hopf bifurcations

It is easy to see from the normalized condition (1.2) on the kernel function that system (1.3) has two feasible boundary equilibria $E_0(0, 0)$, $E_1(\frac{r_1}{a_{11}}, 0)$ and a unique positive equilibrium $E(x^*, y^*)$ when the condition

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