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Complex nonlinear dynamics and controlling chaos in a Cournot duopoly economic model

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ABSTRACT

Complex nonlinear economic dynamics in a Cournot duopoly model proposed by M. Kopel is studied in detail in this work. By utilizing the topological horseshoe theory proposed by Yang XS, the authors detect the topological horseshoe chaotic dynamics in the Cournot duopoly model for the first time, and also give the rigorous computer-assisted verification for the existence of horseshoe. In the process of the proof, the topological entropy of the Cournot duopoly model is estimated to be bigger than zero, which implies that this economic system definitely exhibits chaos. In particular, the authors observe two different types of economic intermittencies, including the Pomeau–Manneville Type-I intermittency arising near a saddle-node bifurcation, and the crisis-induced attractor widening intermittency caused by the interior crisis, which lead to the appearance of intermittency chaos. The authors also observe the transient chaos phenomenon which leads to the destruction of chaotic attractors. All these intermittency phenomena will help us to understand the similar dynamics observed in the practical stock market and the foreign exchange market. Besides, the Nash-equilibrium profits and the chaotic long-run average profits are analyzed. It is numerically demonstrated that both firms can have higher profits than the Nash-equilibrium profits, that is to say, both of the duopolists could be beneficial from a chaotic market. The controlled Cournot duopoly model can make one firm get more profit and reduce the profit of the other firm, and control the system to converge to an equilibrious state, where the two duopolists share the market equally.

1. Introduction

Ever since Day introduced nonlinear dynamics into economics [1], great efforts have been made to study the complex nonlinear behavior in the economic models and practical economic data, including the chaotic dynamics. The theory of chaos in economics has brought valuable insights about how economic systems behave and help people to deeply understand some economic phenomena. It has been shown that many kinds of economic markets can be chaotic. For example, Boldrin and Woodford summarized that the competitive market could be chaotic [2]. Puu studied the adjustment process of two or three oligopolists with an iso-elastic demand function and constant marginal costs, and showed that the system can result in periodic and chaotic behavior [3–5]. This finding implies that even the oligopolistic market could become chaotic. Kopel also investigated the complex adjustment dynamics in some Cournot duopoly models and obtained similar results [6]. Many other economic models have been verified to have chaotic dynamics, such as Goodwin growth model, cobweb price

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adjustment process, Keynesian business cycle model, competitive model with generic and brand efforts [7] and so on. Although these works have shown that economic models could be chaotic, to our best knowledge, researchers always analyzed chaotic behavior by numerical simulations and few has given the theoretical proof for the existence of chaos in economic models.

The Ši'lnikov method is the most commonly used method to prove the existence of chaos rigorously in a mathematical manner. The Melnikov method is the next most common approach. For example, a rigorous mathematical proof for the existence of Chen's attractor was reported by applying the Ši'lnikov criterion [8], and a SIR model of epidemic dynamics was proved to possess chaos by Melnikov's method [9]. But those two methods both require tremendous amount of complicated algebraic derivations. However, the newly proposed topological horseshoe theory [10] provides a simpler and more convenient method to prove the existence of chaos in a computer-assisted manner. According to the chaos theory, the existence of horseshoe in dynamical systems may be the most essential characteristic of chaos. Smale constructed the first horseshoe map, namely the famous Smale horseshoe map, but the conditions of existence of Smale horseshoe are too strict. In recent years, Yang XS has presented a famous topological horseshoe theorem [10,11], which is more practical to be applied to many dynamical systems. It is a computer-assisted proof method, by utilizing which, lots of famous chaotic systems have been verified to have topological horseshoe dynamics, such as the Lorenz system [12], the Ikeda map [12], the Rössler system [13], the cellular neural networks [14] and so on.

In this work, we will study one of the Cournot duopoly models proposed by Kopel [6], and try to go one step further to give a rigorous computer-assisted verification for the existence of horseshoe chaos in this model. For this discrete nonlinear map, we find a proper Poincaré map, under which there exists a closed invariant set that is taken as the field of definitions to make the proper map semi-conjugate to 2-shift map. This implies that the topological entropy is bigger than zero, and the duopoly model definitely exhibits chaotic dynamics.

Besides, the intermittency phenomena in the Cournot duopoly model are also studied in detail. It is known that intermittency is a fundamental feature of complex economic systems. An intermittent time series is characterized by the switching between periodic laminar state and chaotic burst state, or the switching between weak chaotic laminar state and strong chaotic burst state which is larger in the phase–space extent. Previous studies have concluded that there are several different styles of intermittency, for example the Pomeau–Manneville intermittency [15] which includes three different types (type-I, II and III) and the crisis-induced intermittency [16] which includes two different types (attractor widening and attractor merging). In Pomeau–Manneville intermittency, the system seems to switch between periodic or quasiperiodic behavior and chaotic behavior when it is close to a saddle-node bifurcation (type-I), Hopf bifurcation (type-II) and inverse period-doubling bifurcation (type-III) as one parameter varies. However, in the crisis-induced intermittency, two or more chaotic attractors merge to form a single chaotic attractor caused by merging crisis, or a chaotic attractor suddenly changes its size because of interior crisis. Evidence has been found that intermittency is ubiquitous in practical complex economic activities, as well as in the financial and economic models. For example, Mantegna and Stanley [17] explored the possibility that scaling phenomena occur in economic systems, especially when the economic system is one subject to precise rules, as is the case in financial markets. They specifically showed that the scaling of the probability distribution of a particular economic index, namely the Standard & Poor's 500, can be described by a non-Gaussian process with dynamics corresponding to that predicted for a Lévy stable process. Müller et al. [18] presented a statistical analysis of four foreign exchange spot rates against the US Dollar, and pointed out that the mean absolute changes of logarithmic prices follow a scaling law. More recently, Chian et al. chose a forced van der pol oscillator model of economic cycles as the prototype model to study its complex economic dynamics and discussed the type-I and crisis-induced intermittency phenomena [19–21].

One important aim of this paper is to study the economic intermittency in this Cournot duopoly model. We observe the Pomeau–Manneville type-I intermittency, transient chaos and the attractor widening crisis-induced intermittency. The time series, phase portraits and statistical analysis are performed to verify these intermittency phenomena. In Cournot duopoly model, the intermittency phenomenon may imply that in the process of competition, the complex economic system has the ability of self-regulation. The economic system cannot keep steady forever, but once the system becomes chaotic (strongly chaotic), it can make adjustment itself without extra force, and make the system return to the steady state (weak chaos).

In spite of the developments mentioned above, for a duopoly economic system, the duopolists' fundamental concern is the profit they could get. Therefore, a natural question may arise that whether the duopolist could get more profit in chaotic and the intermittency chaotic market or not. In traditional economics, unstable fluctuations have been considered unfavorable, however, Matsumoto recently revealed that chaotic dynamics may be profitable from the long-run perspective [22,23]. Both the Nash-equilibrium profits and the long-run average profits of the Cournot duopoly model we concerned in this paper are analyzed, and we obtain the result that both duopolists could be beneficial from a chaotic market in this Cournot duopoly model. Putting a simple state feedback control makes the system converge to a stationary state, at the same time, one duopolist gets more profit and the other gets less.

The paper is organized as follows. Section 2 introduces the Cournot duopoly model developed by Kopel. Section 3 examines the complex nonlinear economic dynamics, including the bifurcation analysis, the coexistence of two chaotic attractors, the Pomeau–Manneville type-I intermittency, transient chaos and the attractor widening crisis-induced intermittency. Section 4 deals with the long-run average profit analysis and Section 5 considers controlling chaos by a state feedback controller. Section 6 provides the conclusions.